Does Model Uncertainty Justify Conservatism? Robustness and the Delegation of Monetary Policy

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Abstract: This paper analyzes the rationale for delegating monetary policy to an inflation-averse central banker in an environment where the central bank has a preference for robustness of optimal policy with respect to misspecifications of the underlying model of the economy. We use a simple New Keynesian model to show how the optimal output gap weight in the central bank's objective function depends on the degree of model uncertainty. In particular, we show numerically that the rationale for appointing a conservative central bank prevails even if the central bank is concerned about the robustness of policy with respect to model misspecifications. Moreover, we find that the central bank should put more relative weight on inflation stabilization if the degree of uncertainty increases. Interestingly, if the degree of uncertainty is large, monetary policy should be delegated to a conservative central banker even in the absence of shock persistence.

Keywords: optimal monetary policy, delegation, robust control, New Keynesian model, stabilization bias

JEL classification: E32, E52, E58

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1 Introduction

Uncertainty is now recognized to be of central importance for the design of monetary policy.[2] In this paper, we analyze optimal monetary policy in an economy that is plagued by uncertainty about the basic structure of the underlying economic model. That is, the policymaker is not only uncertain about particular parameters and certain data series, but faces uncertainty about the model itself on which he bases optimal policy.[3] In particular, this paper evaluates monetary delegation under model uncertainty, that is, the question of delegating monetary policy to a central bank, whose preferences with respect to the relative weights assigned to output and inflation stabilization differ from that of the social planner.

Since Rogoff's (1985) seminal contribution, it is now common wisdom that delegating monetary policy to a central bank which is more inflation-averse than the social planner, i.e. to a "conservative central banker", can raise welfare. In the class of models Rogoff had in mind, a conservative central banker corrects the problem of an average *inflation bias*. In a more recent generation of micro-founded general equilibrium models for monetary policy analysis, the inflation bias is absent. Nevertheless, these models still motivate the appointment of a conservative central banker. The reason is that monetary policy under discretion gives rise to inefficient inflation stabilization - a *stabilization bias* emerges - which can be corrected by a hawkish central banker. Hence, the case for central bank conservatism is still compelling, although the basic rationale has changed.

In a series of papers, Giannoni (2001, 2002) asks: "Does model uncertainty justify caution?". He analyzes whether the Brainard (1967) result carries over to robust policy in a New Keynesian model of monetary policy.[4] Brainard argued that multiplicative uncertainty should lead to attenuated adjustment of the policy instrument. Giannoni shows that model uncertainty does no longer justify a cautious monetary policy response.[5]

[2] Researchers routinely refer to Alan Greenspan's quote from the 2003 Jackon Hole meeting, which has gained widespread attention: "Uncertainty is not just an important feature of the monetary policy landscape; it is the defining characteristic of that landscape."

[3] For a survey of optimal monetary policy under various dimensions of uncertainty see Walsh (2004a).

[4] Tetlow and von zur Mühlen (2001) find that uncertainty alone cannot explain the observed attenuated policy. Sack (2000) provides empirical evidence of the impact of additive and multiplicative uncertainty on the interest rate setting behavior of the Fed.

[5] Giannoni (2001,2002) does not apply Hansen-Sargent robust control approaches to derive the optimal policy rule. However, as Walsh (2004b) shows, the resulting rule is equivalent to a rule under Hansen-Sargent robustness.

In this paper we revisit another prominent and influential result of the literature, which shaped both the academic thinking about monetary policy and the actual central banking landscape very much like Brainard's finding did. The question is: "Does model uncertainty justify conservatism?".[6] We assess how Rogoff's delegation of monetary policy to a conservative central banker is affected by uncertainty about the underlying model. Does model uncertainty strengthen the case for conservatism or does uncertainty give rise to a less inflation-averse - a liberal - central banker?

We approach model uncertainty within the robust control framework laid out by Hansen and Sargent (2003, 2005). In this environment, the policymaker is unable to formulate a probability distribution over a range of plausible models. A situation in which the policymaker cannot assign probabilities to alternative models is known as Knightian uncertainty following the work of Knight (1921). Here, the central bank desires to design a policy rule that performs well even if the worst possible outcome realizes. In other words, his policy rule should be robust to deviations from his reference model. Central to Hansen and Sargent's robust control approach is the distinction between the policymaker's reference model and the approximating model. The reference model provides the most likely description of the economy. In the absence of model misspecifications, this model generates the conventional rational expectations solution. Under robust control, however, the policymaker believes the model to be misspecified to a certain degree. He formulates a policy rule which is robust to these model distortions and shields the economy from the worst possible misspecification. The approximating equilibrium results if the central bank follows its robust policy in the undistorted reference model. In contrast, the worst case equilibrium corresponds to the case in which the central bank applies the robust rule in the fully distorted model.

Applying robust control techniques provides a convenient platform to analyze model uncertainty, since uncertainty manifests in one additional parameter, the possible amount of misspecification the metaphorical evil agent has available. We can easily reformulate the resulting min-max problem in order to apply standard solution methods for rational expectations models.

Two recent papers examine the delegation problem in related models. Kilponen (2003) analyzes weight-conservatism in a robust control environment. However, he uses a backward-looking model with an average inflation bias generated by an overly ambi-

[6] Blinder (1999, p. 46) argues that "... in the real world, the noun 'central banker' practically cries out for the adjective 'conservative' ".

tious central bank that wants to push output above its natural rate. He finds that the larger the model misspecifications, the more inflation-averse the central banker should be. Gaspar and Vestin (2004) also analyze the rationale for delegation under uncertainty. However, they assume that the central bank knows the true structural relationships of the underlying model but cannot reliably observe potential output. Moreover, they also include endogenous private sector expectations through learning. The central finding is that "the optimal delegation degree of conservatism increases as the quality of knowledge and information declines". In contrast to the first paper, we examine the delegation problem in a small forward-looking monetary model which constitutes the benchmark New Keynesian framework. In contrast to the latter paper, we embed the question in a robust control framework, in which the central bank faces uncertainty not only about a particular data series, but about the model structure as a whole.

We find that the optimal weight attached to output gap fluctuations is lower than society's but greater than zero. Moreover, the degree of central bank conservatism increases with the degree model of uncertainty, that is, with the variety of model misspecifications against which the central banker wants to be robust. The logic behind these results stems from the effect of model uncertainty on the variances of output and inflation. If uncertainty increases inflation variance, the case for appointing a conservative central banker is strengthened, since the stabilization bias is aggravated. Interestingly, if uncertainty is high, monetary policy should be delegated to a conservative central banker even in the absence of persistence, i.e. even when the standard forward-looking model constitutes no case for monetary conservatism. Hence, this paper suggests an alternative rationale for monetary delegation.

This paper is organized as follows. The next section briefly introduces the reference model, which the central bank considers as being the most likely description of the economy. Section 3 formulates the discretionary optimization problem under robustness, while section 4 evaluates the policy outcomes under different degrees of conservatism and uses the social planner's welfare function to assess the optimal degree of inflation-aversion of the central banker. Finally, section 5, summarizes and draws some conclusions.

2 A simple New Keynesian framework

We adopt a standard forward-looking monetary model of the business cycle.[7] The IS curve (1) and the forward-looking Phillips curve (2) represent log-linearised equilibrium conditions of a simple sticky-price general equilibrium model

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \sigma^{-1}\left(i_t - E_t \pi_{t+1}\right) + u_t && (1)\\
\pi_t &= \beta E_t \pi_{t+1} + \gamma y_t + e_t && (2)
\end{aligned}
$$

where π_t is the inflation rate, y_t the output gap, i_t the risk-free nominal interest rate controlled by the central bank, and E_t is the expectations operator. All variables are expressed in percentage deviations from their respective steady state values. The parameters β, γ, and σ are positive and γ depends on the deep parameters of the underlying microeconomic structure. The discount factor is denoted by $\beta < 1$, σ is the elasticity of intertemporal substitution, and γ, the slope coefficient of the Phillips curve, depends negatively on the degree of price stickiness. The processes driving the cost-push shock e_t and the demand shock u_t with standard deviations σ_u and σ_e are given by

$$
\begin{aligned}
u_t &= \rho_u u_{t-1} + \sigma_u \varepsilon_t^u \quad \text{with } 0 \le \rho_u < 1,\ \varepsilon_t^u \sim i.i.d.\,(0,1)\\
e_t &= \rho_e e_{t-1} + \sigma_e \varepsilon_t^e \quad \text{with } 0 \le \rho_e < 1,\ \varepsilon_t^e \sim i.i.d.\,(0,1)
\end{aligned}
$$

This model can be framed in standard state-space form to yield

$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \sigma^{-1} \\ 0 & 0 & 0 & \beta \end{bmatrix}
\begin{bmatrix} u_{t+1} \\ e_{t+1} \\ E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix}
=
\begin{bmatrix} \rho_u & 0 & 0 & 0 \\ 0 & \rho_e & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -\gamma & 1 \end{bmatrix}
\begin{bmatrix} u_t \\ e_t \\ y_t \\ \pi_t \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ \sigma^{-1} \\ 0 \end{bmatrix} i_t
+
\begin{bmatrix} \sigma_u & 0 \\ 0 & \sigma_e \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\begin{bmatrix} \varepsilon_t^u \\ \varepsilon_t^e \end{bmatrix}
\quad (3)
$$

Premultiplying the system with the inverse of the first matrix on the left-hand side of (3) gives the compact conventional notation of forward-looking rational expectations models

$$
\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B} i_t + \mathbf{C}\boldsymbol{\varepsilon}_{t+1} \quad (4)
$$

[7] See, among others, Clarida, Galí, and Gertler (1999) and Woodford (2003) for a deeper analysis and the complete derivation of the model.

where $\mathbf{C}' = [\mathbf{C}_1, \mathbf{0}_{2\times 2}]$ with $\mathbf{C}_1$ denoting the last matrix on the right-hand side of (3). The vector $\mathbf{x}_t' = [\mathbf{x}_{1t}', \mathbf{x}_{2t}']$ summarizes both the state and the jump variables. The 2×1 vector $\mathbf{x}_{1t}$ collects the predetermined variables u_t and $e_t$with $\mathbf{x}_{10}$ given, and $\mathbf{x}_{2t}$ is a 2×1 vector containing the forward-looking variables y_t and π_t. Finally, the 2×1 vector $\boldsymbol{\varepsilon}_{t+1}$ contains the white-noise innovations ε_t^u and ε_t^e.

Monetary policy is assumed to minimize the following loss function, which is quadratic in deviations of the inflation rate, the output gap, and the interest rate from its respective targets, which are set to zero

$$\min_{i_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2\right) \qquad \text{with } \lambda_y, \lambda_i \geq 0 \tag{5}$$

The central bank's loss function differs from the social welfare function only with respect to the weight it attaches to minimizing the variance of the output gap. The central bank penalizes the variance of the output gap with a coefficient λ_y, while society's welfare is calculated using λ_y^{social}. A conservative central banker in the sense of Rogoff (1985) exhibits a smaller weight on y_t^2 than society, i.e. $\lambda_y < \lambda_y^{social}$. The purpose of the paper is to find the optimal value of λ_y and to derive the impact of the degree of model uncertainty on the choice of the optimal output gap coefficient in the central bank's objective function. Note that the weight on inflation stabilization is normalized to one. Hence, the weights on output gap and interest rate stabilization measure the relative emphasis the central bank puts on these conflicting arguments.[8]

3 Optimal robust policy

Minimizing (5) subject to the constraint (4) gives a set of first-order conditions, from which the optimal policy response to shocks can be computed. However, the resulting targeting rule is not robust to misspecifications of the underlying model represented by (1) and (2). To analyze a policy that is robust to model distortions, we apply robust control techniques proposed in a series of contributions by Hansen and Sargent (2003, 2005). The following exposition draws heavily on the survey of Giordani and Söderlind (2004).

The central banker considers the model presented in the previous section as the reference model, which represents the most likely description of the economic structure. However, the policymaker knows that this model could be subject to a wide range

[8] Woodford (2003) shows the conditons under which minimizing a loss function such as (5) corresponds to maximizing the welfare of the representative agent.

of distortions. The task is to reformulate the central bank's optimization problem such that the resulting policy rule performs well even if the model deviates from the reference model. A policy that is optimal in the reference model but does not take account of possible misspecifications can turn out to be disastrous if the misspecifications realize. Under robust control, in contrast, the resulting policy rule performs sufficiently well even if the underlying economic structure does not coincide with the policymaker's reference model.

We transform the minimization problem given by (5) into a min-max problem. The central bank wants to minimize the maximum welfare loss due to model misspecifications by specifying an appropriate policy. To illustrate the problem, we introduce a fictitious second rational agent, the malevolent or evil agent, whose only goal is to maximize the central bank's loss. The evil agent chooses a model from the available set of alternative models and the central bank chooses its policy optimally. Hence, the equilibrium is the outcome of a two-person game. Note that the evil agent is a convenient metaphor for the planner's cautionary behavior. Therefore, the evil agent shares the same reference model that the central bank entertains and optimizes the same objective function. The only difference is that the evil agent wants to maximize rather than minimize the resulting loss.

The set of potential misspecifications, the control vector of the evil agent, takes the form of error terms. However, these shocks are not mere additional exogenous random innovations. Let $\mathbf{v}_{t+1}$ denote the evil agent's (2×1) control vector, which is allowed to feed back on the history of the economy's state variables $\mathbf{x}_t$

$$\mathbf{v}_{t+1} = f_t\left(\mathbf{x}_t, \mathbf{x}_{t-1}, ...\right) \tag{6}$$

where f_t is a sequence of functions. In fact, misspecifications can distort the model parameters, the autocorrelation properties of the error terms, and can introduce non-linearities.[9] The only constraint imposed upon the fictitious evil agent is his budget constraint requiring

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathbf{v}'_{t+1} \mathbf{v}_{t+1} \leq \eta \tag{7}$$

Hence, the parameter η measures the amount of misspecification the evil agent has available.[10] This formulation greatly simplifies the problem, because the degree of

[9] See Hansen and Sargent (2005) for details.

[10] Giordani and Söderlind (2004) note that the evil agent's control vector is indexed $t+1$ although the distortions are known in t. This convention is supposed to stress the fact the the distortions are masked by the shock processes, i.e. the cost-push and the demand shock.

uncertainty manifests in a single parameter. The complete optimization problem thus becomes

$$\min_{\{i\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2\right) \tag{8}$$

$$s.t.\ \ \mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}i_t + \mathbf{C}\left(\boldsymbol{\varepsilon}_{t+1} + \mathbf{v}_{t+1}\right)$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathbf{v}_{t+1}' \mathbf{v}_{t+1} \leq \eta$$

Note that the control variables $\mathbf{v}_{t+1}$ of the evil agent are masked by the shock vector $\boldsymbol{\varepsilon}_{t+1}$. If there were no shocks hitting the economy, the policymaker would be able to perfectly observe model distortions.[11] To simplify the analysis, we assume that the behavior of the evil agent mirrors that of the central bank in that they optimize at the same point in time and play a Nash game.

The standard rational expectations solution for optimal monetary policy corresponds to $\eta = 0$, such that the evil agent's budget is empty. In this case, the maximization part becomes irrelevant. If $\eta > 0$, on the other hand, the central bank faces model distortions. The constraint can be inserted to obtain the Lagrangian

$$\min_{\{i\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 - \theta \left(\mathbf{v}_{t+1}' \mathbf{v}_{t+1}\right)\right] \tag{9}$$

$$s.t.\ \ \mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}i_t + \mathbf{C}\left(\boldsymbol{\varepsilon}_{t+1} + \mathbf{v}_{t+1}\right)$$

The Lagrange parameter θ is inversely related to η. Hence, the rational expectations case corresponds to $\theta \to \infty$.[12] In the following, we will loosely refer to θ as the degree of robustness or the degree of uncertainty, respectively. A lower θ means that the central bank designs a policy which is appropriate for a wider set of possible misspecifications. Therefore, a lower θ is equivalent to a higher degree of robustness.

The loss function and the law of motion for the forward-looking model can be redefined to formulate the optimization program in standard state-space form. This yields

$$\min_{\{i\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left(\mathbf{x}_t' \mathbf{Q} \mathbf{x}_t + \mathbf{h}_t' \mathbf{R} \mathbf{h}_t\right) \tag{10}$$

$$s.t.\ \ \mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \hat{\mathbf{B}}\mathbf{h}_t + \mathbf{C}\boldsymbol{\varepsilon}_{t+1} \tag{11}$$

[11] Intuitively, misspecifications create more damage, if the variance of forecast errors is large.

[12] In this case, the evil agent maximizes the welfare loss by choosing $\mathbf{v}_{t+1} = 0$.

with

$$\mathbf{h}_t' = \begin{bmatrix} i_t & \mathbf{v}_{t+1} \end{bmatrix}, \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} & \mathbf{C} \end{bmatrix}, \mathbf{R} = \begin{bmatrix} \lambda_i & \mathbf{0}_{1\times 2} \\ \mathbf{0}_{2\times 1} & -\theta \mathbf{I}_2 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0}_{2\times 2} & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & \mathbf{Q}_{bb} \end{bmatrix}, \mathbf{Q}_{bb} = \begin{bmatrix} \lambda_y & 0 \\ 0 & 1 \end{bmatrix}$$

Due to the fact that the first order conditions for a minimum are the same as for a maximum, the optimization problem can be solved using standard solution algorithms designed for evaluating optimal policy in rational expectations models.[13]

As in other rational expectations models, the forward-looking variables and the evil agent's and the central bank's control vectors will be linear functions of the predetermined variables in $\mathbf{x}_{1t}$

$$\begin{bmatrix} \mathbf{x}_{2t} \\ i_t \\ \mathbf{v}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{N} \\ -\mathbf{F}_i \\ -\mathbf{F}_{\mathbf{v}} \end{bmatrix} \mathbf{x}_{1t} \tag{12}$$

The equilibrium dynamics of the model are found by combining this solution with the reference model $\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \hat{\mathbf{B}}\mathbf{h}_t + \mathbf{C}\varepsilon_{t+1}$. If the full amount of possible misspecifications realizes, we refer to the resulting model as the worst case model, which is formally obtained by inserting (12) in the law of motion for the reference model

$$\mathbf{x}_{t+1} = \left(\mathbf{A} - \hat{\mathbf{B}}\mathbf{F}_i - \mathbf{C}\mathbf{F}_{\mathbf{v}}\right)\mathbf{x}_t + \mathbf{C}\varepsilon_{t+1} \tag{13}$$

If, on the other hand, the reference model turns out to be undistorted, we refer to the resulting model as the approximating model, which is obtained by setting $\mathbf{F}_{\mathbf{v}} = 0$ in (13)

$$\mathbf{x}_{t+1} = \left(\mathbf{A} - \hat{\mathbf{B}}\mathbf{F}_i\right)\mathbf{x}_t + \mathbf{C}\varepsilon_{t+1} \tag{14}$$

A central bank concerned with robustness designs policy based on the fully distorted model. Once policy is formulated, however, the central bank acts as if there were no longer any model uncertainty. We use the resulting solutions in this approximating equilibrium to evaluate the delegation problem of monetary policy.

4 Delegating monetary policy

In the standard New Keynesian model, the rationale for appointing a conservative central banker is not an average inflation bias generated by an overly ambitious central

[13] In this paper, the solution follows the procedures and the MATLAB software outlined in Söderlind (1999) and Giordani and Söderlind (2004).

banker who wants to steer unemployment below the natural rate. Instead, the motivation for appointing a central banker whose relative weights differ from those of the planner is known as the stabilization bias. Clarida, Galí, and Gertler (1999), among others, have noted that under discretion, inflation is inefficiently stabilized. As a result, the variance of inflation is higher than under the commitment benchmark. Appointing a Rogoff-conservative central banker, who smooths inflation volatility, can raise welfare. Because the public knows inflation will respond less to a cost-push shock, future expected inflation rises less. Stabilizing inflation becomes less costly in terms of future output.[14] Hence, appointing a Rogoff-conservative provides a (second-best) solution to the stabilization bias.[15] Note that this rationale strengthens as the persistence of the cost-push shock process increases. A larger persistence eventually translates into higher volatility and aggravates the stabilization bias. In the standard model without persistence of the cost-push shock, the motivation for conservatism disappears. Below we will see that a high degree of model uncertainty provides a rationale for conservatism even in this case of uncorrelated shocks.

The optimization problem from the perspective of a welfare maximizing social planner is the following. He chooses the optimal weight in the central bank's objective function by maximizing welfare, which is given by the (negative) present-value of the loss function evaluated for a given λ_y

$$\begin{aligned}\max_{\lambda_y} \mathcal{W} &= -(1-\beta) E_0 \sum_{t=0}^{\infty} \beta^t \left(L_t | \lambda_y \right) \qquad (15)\\ &= -(1-\beta) E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\pi_t^2 \middle| \lambda_y \right) + \lambda_y^{social} \left(y_t^2 \middle| \lambda_y \right) + \lambda_i \left(i_t^2 \middle| \lambda_y \right) \right\} \\ &= -\left\{ var \left(\pi_t | \lambda_y \right) + \lambda_y^{social} var \left(y_t | \lambda_y \right) + \lambda_i var \left(i_t | \lambda_y \right) \right\}\end{aligned}$$

Hence, the optimal output weight λ_y of the central bank, which is inversely related to the degree of conservatism, is chosen such that the welfare function is maximized. The government chooses the degree of conservatism given the outcomes $\left(\pi_t^2 \middle| \lambda_y \right)$, $\left(y_t^2 \middle| \lambda_y \right)$, and $\left(i_t^2 \middle| \lambda_y \right)$ that the central banker with λ_y achieves under model uncertainty.

The solution algorithm provides the variances needed to evaluate the welfare function. However, we do not have analytical solutions for the arguments of (15) at hand. There-

[14] Another paper focussing on monetary delegation in a New Keynesian model is Carboni and Ellison (2003). They show that the gain from appointing a conservative central banker may be attenuated or even reversed if the degree of price stickiness becomes endogenous.

[15] See Walsh (2003) for an explicit derivation of the stabilization bias under discretionary optimization in a model similar to that presented above and its possible solution by appointing a conservative central banker.

fore, we compute the welfare loss for a given combination of θ and λ_y and search for the lowest loss over a grid of plausible values of λ_y. The parameter values for calibrating the solutions and the variances of the output gap, the inflation rate, and the interest rate are reported in table (1).

Table 1: Calibrated parameter values

	reference model			loss functions		shock processes		
	β	γ	σ	λ_i	λ_y^{social}	ρ_e	ρ_u	$\sigma_e = \sigma_u$
I	0.99	0.024	0.16	0.00	0.25	0.00	0.00	1
II	0.99	0.024	0.16	0.00	0.25	0.20	0.00	1
III	0.99	0.024	0.16	0.00	0.25	0.50	0.00	1
IV	0.99	0.024	0.16	0.24	0.05	0.00	0.00	1
V	0.99	0.024	0.16	0.24	0.05	0.35	0.00	1

The choice of the discount factor $\beta = 0.99$ is standard in the literature and is consistent with an annual real interest rate of 4 percent. All other model parameters are set following the work of e.g. Walsh (2004a) and Giannoni (2002), i.e. $\gamma = 0.024$ and $\sigma = 0.16$. The first three specifications exhibit no interest rate stabilization objective, i.e. $\lambda_i = 0$. With $\lambda_y^{social} = 0.25$, we employ a both plausible and widely used calibration. In specifications IV and V, we apply the calibration of Giannoni (2002), Giannoni and Woodford (2003), and Woodford (2003), which implies a large penalty for interest rate variation with an output gap weight of only 0.05. The shock processes are also standard.

We evaluate the welfare loss and the rationale for appointing a conservative central banker under model uncertainty over a grid of plausible values of the robustness parameter θ. This parameter, however, is bounded only by $\theta > 0$ with rational expectations corresponding to $\theta \to \infty$. Hence, we have no reasonable a priori range over which we should compute welfare. To overcome the problem of specifying a range for θ, we follow Hansen and Sargent (2005, chapter 8) and employ what they refer to as a detection error probability approach. These authors use statistical theory to formulate a probability for discriminating between the approximating model and the worst case model and to obtain a context-specific θ. This approach, which is calculated using likelihood-ratio tests, is based on the idea that the alternative models the central bank faces should not be easily distinguishable with a reasonable set of data. Zero robustness, i.e. the rational expectations case, corresponds to a probability of 0.5. We calculate this probability and invert it to obtain a value of θ. The resulting relationship between detection probabilities and θ is presented in figure (1). A range of detection error probabilities from zero to 0.5 corresponds to values of θ ranging roughly from 25 to 200. We choose this range as a guideline in specifying the grid of robustness parameters over which we will evaluate the welfare implications of monetary policy delegation.[16]

5 Results

The results for the five alternative sets of parameter values are presented in tables (2) to (6) in the appendix. Each cell gives the value of the welfare function for a combination of θ and λ_y. For each degree of robustness, the output weight that generates the lowest welfare loss is printed in bold. Moreover, the results of model V are visualized in figure (2) for a larger grid than reported in the table. A first glance at this figure reveals that the degree of model uncertainty, measured by the (the inverse of) θ has a strong impact on social welfare. The lower θ, the more sharply welfare deteriorates. Four results stand out:

> **Result 1:** The welfare gain from delegating monetary policy to a policymaker with the optimal output weight increases as the uncertainty over the true model of the economy grows.

Since the level of the welfare loss in absolute terms contains no direct economic meaning, we apply Jensen's (2002) metric to express the welfare effect of appointing a central banker with an optimal degree of conservatism in terms of an equivalent permanent increase in inflation, the so-called "inflation equivalent". The inflation equivalent π^{equiv} describes a permanent deviation of inflation from target that in welfare terms is equivalent to delegating monetary policy optimally. This inflation equivalent is given by

$$\pi^{equiv} = \sqrt{\mathcal{W}|\,\lambda_y^{opt} - \mathcal{W}|\,\lambda_y^{social}} \tag{16}$$

[16] This is consistent with, among others, the work of Dennis (2005), who specifies θ to be equal to 500, 100, or 50.

Note that a permanent deviation of inflation from target of π^{equiv} percent results in an increase in the objective function by $E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi^{equiv}\right)^2$. Thus, the inflation equivalent satisfies

$$\mathcal{W}|\,\lambda_y^{opt} + \pi^{equiv} = \mathcal{W}|\,\lambda_y^{social} \tag{17}$$

We find that π^{equiv} increases if θ falls. A higher degree of model uncertainty makes the delegation of monetary policy more important.

> **Result 2:** The optimal output weight of the central bank is equal or smaller than the social planner's weight. Hence, there is a case for a conservative central banker. In all specifications, the degree of optimal conservatism increases, if monetary policy becomes more robust.

In all specifications, the parameter λ_y^{opt} that leads to the lowest value of the welfare loss fulfills $\lambda_y^{opt} \leq \lambda_y^{social}$. Hence, delegating monetary policy to a policymaker who puts less weight on output gap stabilization and relatively more weight on inflation stabilization - a Rogoff-type conservative central banker - increases welfare. Under $\theta = 50$ in scenario II, for example, $\lambda_y^{opt} = 0.15 < 0.25$. The reason is that a central banker that puts more weight on inflation stabilization reduces inflation volatility and therefore mitigates the stabilization bias of optimal monetary policy in the absence of commitment. Remember that the model provides a rationale for appointing conservative central banker even in the absence of an average inflation bias. The degree of conservatism increases if we set the persistence of the cost shock to 0.20 or 0.50, see tables (3) and (4). The reason is that more persistent shocks raise the variance of the inflation rate and, hence, aggravate the stabilization bias.

How does the degree of robustness affect the optimal delegation? Consider specifications I to V. If the central bank becomes more robust, i.e. if the range of model misspecifications increases against which the central bank wants to immunizes her policy, the optimal output weight falls. In other words, the optimal degree of conservatism increases as model uncertainty increases. If the degree of robustness increases from $\theta = 100$ to $\theta = 10$ in specification II, λ_y^{opt} falls from 0.20 to 0.10. The mechanism behind this finding is the fact that an increase in robustness amplifies inflation volatility. Consider for example specification V and fix λ_y at 0.05. While $\theta = 200$ generates an inflation variance of 2.44, moving to a more robust policy with $\theta = 60$ raises inflation volatility to 2.98.[17] Under these circumstances, an increased preference for robustness

[17] These results corroborate the finding of Giordani and Söderlind (2004), who argue that robustness increases the volatility of inflation and the output gap.

strengthens the case for delegating policy to a Rogoff-conservative. Figure (2) illustrates this finding. Those values for λ_y that maximize welfare clearly fall as robustness increases, i.e. as θ decreases.

> **Results 3:** Even in the absence of persistence, monetary policy should be delegated to a conservative central banker, if the degree of uncertainty is large.

Even if the persistence of shocks is zero, i.e. even if the standard New Keynesian model under rational expectations provides no rationale for a conservative central banker, a high degree of robustness requires a weight on output gap stabilization lower than that of the social planner.[18] Hence, high model uncertainty provides an alternative motivation for delegating monetary policy even in the absence of persistence. Consider specification IV in table (5) as an example. Under $\theta = 100$, the optimal output weight corresponds to that of the social planner, i.e. $\lambda_y^{opt} = \lambda_y^{social} = 0.05$. If the degree of robustness increases to $\theta = 20$, $\lambda_y^{opt} = 0.04 < \lambda_y^{social}$. All results would, in general, be quantitatively much more pronounced if we set the variance of cost-push shocks to values higher than unity. The qualitative results, however, remain unchanged.

> **Result 4:** Appointing a central banker who is too conservative, i.e. $\lambda_y < \lambda_y^{opt}$, is more costly in term of welfare than appointing a central banker who is too liberal, i.e. $\lambda_y > \lambda_y^{opt}$.

Consider for example specification III in table (4) under $\theta = 150$. Moving from the optimal output weight of 0.10 to a slightly more conservative central banker with $\lambda_y = 0.05$ entails a much larger additional welfare loss than moving to a slightly more liberal central banker with $\lambda_y = 0.15$.[19] Figure (2) illustrates that welfare sharply deteriorates if λ_y lies below its welfare maximizing value but only mildly decreases if λ_y lies above the optimal weight.

6 Conclusions

This paper contributes to the growing literature on optimal monetary policy under model uncertainty. Specifically, we analyzed which relative weight the central bank

[18] Equivalently, if the persistence of cost-push shocks is zero, there is a case for a conservative central banker even in the absence of inertia in the inflation adjustment equation (2) as long as uncertainty is sufficiently high.

[19] This corresponds to the findings of Gaspar and Vestin (2004).

should optimally attach to fluctuations in the output gap as opposed to fluctuations in inflation and the interest rate. We found that the presence of model uncertainty makes this question even more important as the optimal degree of conservatism, i.e. the relative weight on inflation stabilization, increases with growing uncertainty. Hence, this paper showed that the seminal result of Rogoff (1985) - that a central banker could be optimal which places a larger weight on inflation stabilization than the social planner - still holds under uncertainty about the structure of the economy. This paper answered this question in a standard forward-looking monetary policy model without an average inflation bias and used robust control techniques to derive optimal policy. Given that model uncertainty and the desire for robustness are among the striking features of contemporary monetary policy, these results lend further support to the robustness of the classic Rogoff-result itself.
This paper leaves a number of questions unanswered which affect optimal monetary policy under model uncertainty and, in particular, the design of monetary institutions. For example, the social planner and the central bank are assumed to share the same degree of robustness. It might be interesting to allow for discrepancies in the awareness with respect to model uncertainty. Furthermore, this paper follows the literature and treats the degree of robustness as an exogenous parameter. A fruitful extension should explore the trade-off that could determine an optimal degree of robustness. We leave these questions for further research.

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Table 2: Welfare in specification I

output weight	$\rho_e = 0.00, \lambda_y^{social} = 0.25, \lambda_i = 0.00$					
	degree of robustness					
λ_y	θ =200	θ =150	θ =100	θ =50	θ =25	θ =10
0.05	-103.321	-103.661	-104.357	-106.678	-111.766	-140.651
0.10	-100.256	-100.592	-101.279	-103.476	-108.621	-137.827
0.15	-99.852	-100.189	-100.877	-103.079	-108.241	**-137.741**
0.20	-99.768	-100.105	-100.795	-103.003	**-108.179**	-137.864
0.25	**-99.755**	**-100.093**	**-100.784**	**-102.996**	-108.183	-137.991
0.30	-99.763	-100.101	-100.793	-103.008	-108.203	-138.099
π^{equiv}	0.000	0.000	0.000	0.000	0.006	0.050

Notes: The table reports welfare (multiplied by 100). The minimum welfare loss for each robustness scenario is given in bold. The welfare gain from optimal monetary delegation is reported in terms of Jensen's (2002) inflation equivalent π^{equiv}.

Table 3: Welfare in specification II

	$\rho_e = 0.20, \lambda_y^{social} = 0.25, \lambda_i = 0.00$					
output weight	degree of robustness					
λ_y	θ =200	θ =150	θ =100	θ =50	θ =25	θ =10
0.05	-168.532	-169.884	-172.727	-182.638	-213.662	-277.357
0.10	-164.041	-165.386	-168.218	-178.125	-209.605	**-270.786**
0.15	-163.551	-164.901	-167.747	**-177.713**	**-209.546**	-271.683
0.20	**-163.499**	**-164.854**	**-167.710**	-177.717	-209.768	-272.810
0.25	-163.530	-164.888	-167.751	-177.787	-209.982	-273.717
0.30	-163.576	-164.937	-167.806	-177.863	-210.159	-274.419
π^{equiv}	0.018	0.018	0.020	0.027	0.066	0.171

Notes: The table reports welfare (multiplied by 100). The minimum welfare loss for each robustness scenario is given in bold. The welfare gain from optimal monetary delegation is reported in terms of Jensen's (2002) inflation equivalent π^{equiv}.

Table 4: Welfare in specification III

output weight	$\rho_e = 0.50, \lambda_y^{social} = 0.25, \lambda_i = 0.00$					
	degree of robustness					
λ_y	θ =5000	θ =1000	θ =500	θ =200	θ =150	θ =100
0.04	-534.071	-544.602	-558.987	613.541	-655.662	-809.616
0.05	-524.029	-534.532	-548.900	-603.639	-646.243	**-806.644**
0.10	-514.104	-524.753	**-539.364**	**-595.573**	**-640.072**	-819.889
0.15	**-513.933**	**-524.697**	-539.482	-596.548	-641.999	-830.888
0.20	-514.456	-525.291	-540.181	-597.753	-643.747	-837.804
0.25	-514.967	-525.850	-540.809	-598.709	-645.651	-842.439
π^{equiv}	0.102	0.107	0.120	0.177	0.236	0.598

Notes: The table reports welfare (multiplied by 100). The minimum welfare loss for each robustness scenario is given in bold. The welfare gain from optimal monetary delegation is reported in terms of Jensen's (2002) inflation equivalent π^{equiv}.

Table 5: Welfare in specification IV

output weight	$\rho_e = 0.00, \lambda_y^{social} = 0.05, \lambda_i = 0.24$					
	degree of robustness					
λ_y	θ =200	θ =100	θ =50	θ =30	θ =20	θ =10
0.01	-105.470	-106.437	-108.520	-111.659	-116.354	-142.359
0.02	-100.847	-101.853	-104.023	-107.308	-112.252	**-140.582**
0.03	-99.845	-100.867	-103.073	-106.419	-111.167	-140.811
0.04	-99.586	-100.616	-102.841	-106.217	**-111.318**	-141.212
0.05	**-99.542**	**-100.576**	**-102.811**	**-106.205**	-111.338	-141.573
0.06	-99.568	-100.605	-102.847	-106.252	-111.405	-141.872
π^{equiv}	0.000	0.000	0.000	0.000	0.014	0.099

Notes: The table reports welfare (multiplied by 100). The minimum welfare loss for each robustness scenario is given in bold. The welfare gain from optimal monetary delegation is reported in terms of Jensen's (2002) inflation equivalent π^{equiv}.

Table 6: Welfare in specification V

output weight λ_y	$\rho_e = 0.35, \lambda_y^{social} = 0.05, \lambda_i = 0.24$ degree of robustness θ =200	θ =150	θ =100	θ =80	θ =75	θ =60
0.004	-289.470	-194.861	-307.246	-318.393	-322.574	-342.545
0.005	-287.420	-292.972	-305.747	-317.266	-321.591	**-342.300**
0.006	-286.106	-291.788	-304.873	-316.687	-321.127	-342.423
0.007	-285.255	-291.041	-304.377	-316.430	**-320.964**	-342.733
0.008	-284.702	-290.575	-304.117	**-316.367**	-320.977	-343.137
0.009	-284.349	-290.294	-304.008	-316.422	-321.096	-343.581
0.010	-284.131	-290.137	**-303.997**	-316.549	-321.276	-344.036
0.011	-284.007	-290.064	-304.048	-316.718	-321.492	-344.487
0.012	-283.948	**-290.050**	-304.141	-316.913	-321.726	-344.924
0.013	**-283.935**	-290.076	-304.259	-316.120	-321.967	-345.343
0.014	-283.954	-290.129	-304.394	-317.332	-322.210	-345.741
π^{equiv}	0.154	0.167	0.198	0.230	0.242	0.311

Notes: The table reports welfare (multiplied by 100). The minimum welfare loss for each robustness scenario is given in bold. The welfare gain from optimal monetary delegation is reported in terms of Jensen's (2002) inflation equivalent π^{equiv}.

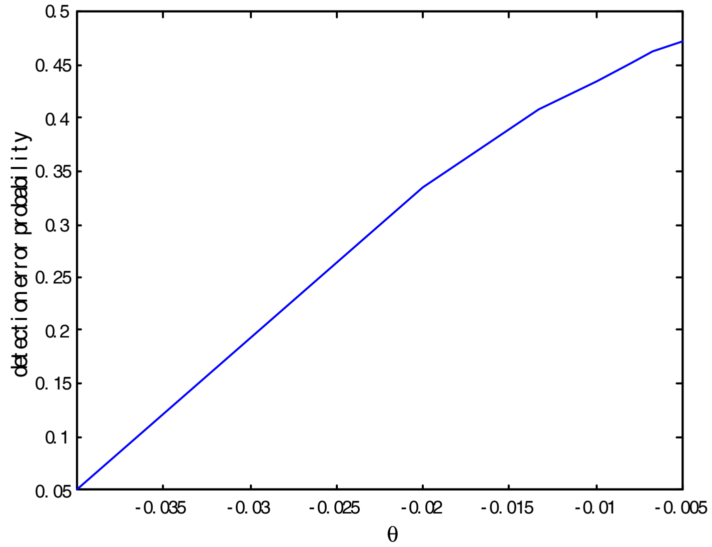

Figure 1: Detection error probabilities for different values of $-\theta^{-1}$ under parameter specification IV

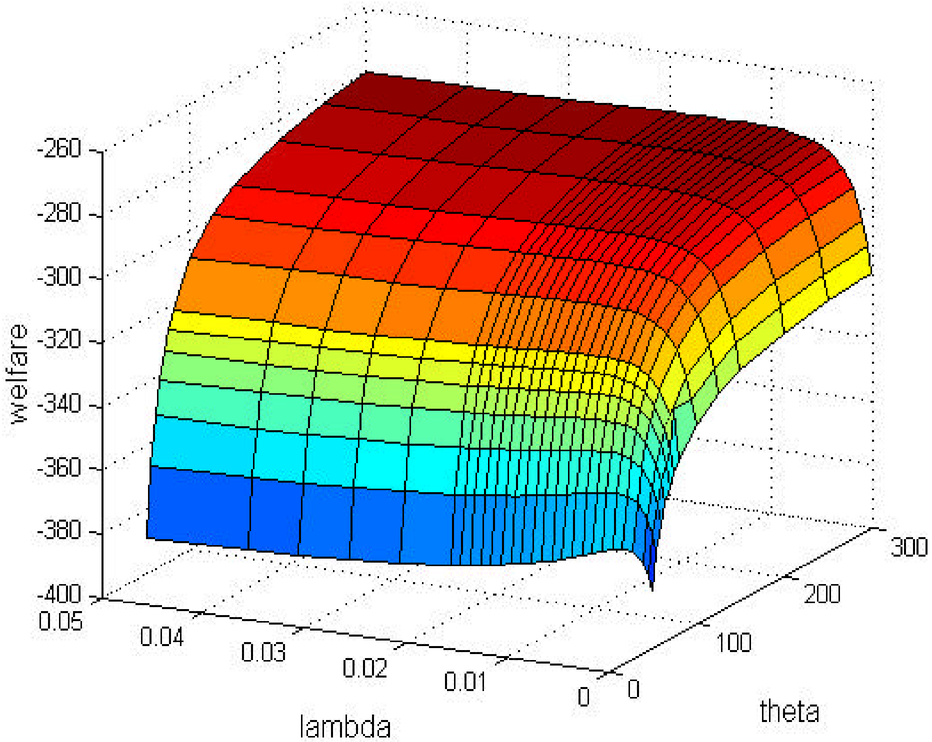

Figure 2: Welfare, robustness θ, and the output weight λ_y in specification V