The New Keynesian Phillips Curve in Europe: does it fit or does it fail?

Peter Tillmann[1]
University of Bonn
Institute for International Economics
Lennéstr. 37, D-53113 Bonn
tillmann@iiw.uni-bonn.de

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Abstract: The canonical New Keynesian Phillips curve specifies inflation as the present-value of future real marginal costs. This paper exploits projections of future real marginal costs generated by VAR models to assess the model's ability to match the behavior of actual inflation in the Euro area. The model fits the data well at first sight. A set of bias-corrected bootstrapped confidence bands, however, reveals that this result is consistent with both a well fitting and a failing model.

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1 Introduction

Under the New Keynesian paradigm, inflation dynamics are forward-looking. The workhorse New Keynesian Phillips Curve (NKPC) relates current inflation to expected future inflation and current real marginal costs. Equivalently, the inflation rate is

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given as the present-value of the entire expected path of future real marginal costs. This present-value relation implicit in any off-the-shelf New Keynesian model is the central topic of this paper as it lends itself to a well-established empirical approach. Specifically, we can employ forecasts from a Vector Autoregression (VAR) to generate a series of model-consistent or "fundamental" (Galí, Gertler, and López-Salido 2001) inflation that can be contrasted with actually observed inflation.

In this paper we assess the empirical fit of the present-value relation implied by the Calvo (1983) price setting scheme using data for the aggregate Euro area. In particular, we follow Kurmann (2005) and take account of estimation uncertainty. In contrast to Galí et al. (2001) we find that the forward-looking NKPC fits Euro data only at first sight. Huge confidence bands preclude an interpretation of conventionally employed measures of fit.

The present paper is organized as follows. The next section briefly presents the present-value relation for inflation. Section 3 presents the estimation strategy and elaborates the bootstrap approach to calculate confidence intervals around standard measures of fit. Section 4 presents the results and, finally, section 5 concludes.

2 The New Keynesian model of inflation

Consider the case of staggered price setting following Calvo (1983). Each firm adjusts its price during the current period with a fixed probability $1-\mu$, where $0<\mu<1$. It can be shown that inflation is determined by expected future inflation and current real activity proxied by real marginal costs, where π_t is the inflation rate, rmc_t denotes a measure of real marginal costs, and E_t is the expectations operator

$$\pi_t = \beta E_t \pi_{t+1} + \gamma rmc_t \tag{1}$$

The composite slope-coefficient γ is given by $\frac{(1-\mu)(1-\beta\mu)}{\mu}$ and the discount factor is given by $\beta<1$. Repeated substitution then yields

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t rmc_{t+k} \tag{2}$$

Equation (2) says that the inflation rate at time t is a fraction of the present-value of the expected path of future real marginal costs.

3 The present-value relation under estimation uncertainty

To assess the explanatory power of the New Keynesian Phillips curve, we construct an implied series for the forward-looking terms and contrast model-consistent inflation rates with actually observed inflation rates following the approach of Campbell and Shiller (1987). Let the information set of agents be described by past realizations of inflation and real marginal costs. The vector $\mathbf{Z}_t = [rmc_t, ..., rmc_{t-q+1}, \pi_t, ..., \pi_{t-q+1}]'$ follows a VAR(q) in companion form

$$\mathbf{Z}_{t+1} = \mathbf{A}\mathbf{Z}_t + \mathbf{\Gamma}_{Z_{t+1}} \tag{3}$$

where $\mathbf{\Gamma}_{Z_{t+1}} = [u_{1t}, 0, ..., 0, u_{2t}, 0, ...0]'$ represent innovations to agents' information sets and $\mathbf{A}$ is the $2q \times 2q$ matrix . We will later check for the robustness of the results and will use the alternative forecasting VAR with $\mathbf{Z}_t = [rmc_t, ..., rmc_{t-q+1}, \Delta nmc_t, ..., \Delta nmc_{t-q+1}]'$, i.e. a VAR with current and lagged realizations of the level of real marginal costs and changes of nominal marginal costs. To check the robustness of the results, we will also estimate a three-dimensional VAR system than contains the short-term interest rate as an additional variable.

Forecasts based on the econometrician's information set $\mathbf{H}_t$, which includes only current and lagged values of the variables in $\mathbf{Z}_t$, are given by the multi-period forecasting formula

$$E_t [\mathbf{Z}_{t+k} | \mathbf{H}_t] = \mathbf{A}^k \mathbf{Z}_t \tag{4}$$

The vector of the discounted future paths of the variables can be calculated using the summation formula for infinite geometric series

$$\sum_{k=0}^{\infty} \beta^k E_t \mathbf{Z}_{t+k} = (\mathbf{I} - \beta \mathbf{A})^{-1} \mathbf{Z}_t \tag{5}$$

We map these forecasts into the present-value representation of the Calvo pricing model to obtain an expression for the model-consistent inflation rate. This theoretical or "fundamental" inflation rate is given by

$$\pi_t^{fund} = \gamma \sum_{k=0}^{\infty} \beta^k E_t \{rmc_{t+k}\} = \gamma \mathbf{h}'_{rmc} (\mathbf{I} - \beta \mathbf{A})^{-1} \mathbf{Z}_t \tag{6}$$

where $\mathbf{h}'_{rmc}$ denotes a selection vector that singles out the forecast of real marginal costs, i.e. the first element of $(\mathbf{I} - \beta \mathbf{A})^{-1} \mathbf{Z}_t$. The NKPC thus predicts that inflation at time t should be a scalar multiple of the first entry in the vector $(\mathbf{I} - \beta \mathbf{A})^{-1} \mathbf{Z}_t$. Rudd and Whelan (2003) propose to infer the slope coefficient γ from an OLS regression of actual inflation on the present-value of future real marginal costs $\mathbf{h}'_{rmc}(\mathbf{I}-\beta\mathbf{A})^{-1}\mathbf{Z}_t$ and a constant.

We plot actual inflation against fundamental inflation and compute two standard measures of fit. The first measure is the ratio of standard deviations. The second measure is the correlation coefficient between fundamental and actual inflation. To assess the accuracy of the model's fit to the actual data, we employ a bootstrap approach that infers the distribution of our measures of fit from estimating the model with artificially created data.
We obtain confidence intervals by drawing from the residuals of the estimated VAR model and generating new observations for the $\mathbf{Z}_t$ vector using the estimated companion matrix $\hat{\mathbf{A}}$. Using the artificially created observations, the VAR model is estimated again and a new coefficient matrix is computed. From this we compute the series of expected real marginal costs and regress actual inflation on the present value of future real marginal costs to infer the slope coefficient. Finally, the ratio of standard deviation and the correlation coefficient is computed. Repeating this procedure 10000 times provides us with an empirical distribution for the ratio of standard deviations and the correlation coefficient. The 90% confidence intervals lie between the 5% and 95% percentile endpoints of this distribution.[2]
However, Kilian (1998) shows that this standard bootstrap algorithm performs poorly when it is used to compute distributions of statistics that are nonlinear functions of VAR parameters. Note that both the ratio of standard deviations and the correlation coefficient are indeed highly nonlinear functions of the estimated VAR coefficients. Therefore, we cannot rely on the conventional bootstrap approach here since the small sample distributions of the measures of fit are likely to be biased.
Therefore, we apply Kilian's bias-corrected bootstrap algorithm. Basically, he proposes to replace the estimated VAR coefficients $\hat{\mathbf{A}}$ by bias-corrected estimates $\bar{\mathbf{A}}$ before running the bootstrap to compute the measures of fit. Details about this bias-correction can be found in Kilian (1998) and Kurmann (2005). Moreover, Kilian (1998) proposes a second bias-correction because the OLS estimates are themselves biased away from their population values. We therefore should replace $\hat{\mathbf{A}}$ prior to generating artificial data series. The approach amounts to a bootstrap-after-bootstrap technique. In a first step we do a bootstrap to approximate the OLS small-sample bias. In a second step we replace the coefficients with bias-corrected coefficients, use the first stage bias-

[2]This bootstrap approach has the advantage of respecting the boundedness of the correlation coefficient $-1 < \rho < 1$ and the ratio of standard deviations $\sigma(\pi_t^{fund})/\sigma\left(\pi_t^{actual}\right) > 0$, and allowing for skewness because it does not impose symmetry.

correction again and use a second bootstrap-round to generate the distribution of our estimates and measures of fit.

4 Results

We use quarterly data for the Euro area obtained from the ECB's Area Wide Model database covering 1970:1 -2003:4. Inflation is measured as the first difference of the logarithm of the implicit GDP deflator. Real marginal costs can be shown to be proportional to labor's share of income (in deviations from its mean). Following Coenen and Wieland (2005) and others, we account for the secular downward trend in inflation and the labor share over the sample period by removing a linear time trend prior to estimation.[3]

We proceed by calculating fundamental inflation according to the model laid out before. Following Rudd and Whelan (2003), actual inflation is regressed on the present-value of future real marginal costs in order to infer the parameter γ. We then compare the series of fundamental inflation with actual inflation by means of the ratio of their standard deviations and their correlation coefficient. Under the null hypothesis, $\pi_t^{actual} = \pi_t^{fund}$; hence $\rho\left(\pi_t^{actual}, \pi_t^{fund}\right) = 1$ and $\sigma(\pi_t^{fund})/\sigma\left(\pi_t^{actual}\right) = 1$, where $\rho(\cdot)$ is the correlation coefficient and $\sigma(\cdot)$ is the standard deviation.

The results for the baseline model is presented din table (1) together with several slight modifications and robustness checks. In most specifications, we set the discount factor to 0.99 which is the standard calibration in the literature. Model I, our baseline model, yields a series of fundamental inflation, see figure (1), that, at first sight, tracks the actual European inflation rate quite well. The ratio of standard deviations is 0.70 and the correlation coefficient between actual and fundamental inflation is 0.74, see table (1). However, this impressive fit is merely based on point estimates. The confidence bands obtained from the bootstrap approach reveal that both measures are associated with an extremely large degree of uncertainty. In fact, the confidence band shows that a volatility ratio of 0.34 is as likely (within a 90% band) as a ratio of 1.82. The confidence band around the correlation coefficient is substantially narrower and includes correlations between 0.57 and 0.83. Hence we cannot say whether the model fits or fails. It could equally likely explain 34% of the variation of the inflation rate and more than 150% of the variation in inflation.

[3] We estimate the auxiliary VAR models with five lags in order to minimize serial correlation in the estimated residuals. To check for robustness, we will also present results obtained from a VAR with three lags.

To check for the robustness, models II through VI feature slightly different specifications. Model II is estimated using a lower discount factor of $\beta = 0.98$ and model III includes real marginal costs and changes in nominal marginal costs instead of inflation. These models yield equally wide confidence bands. Model IV includes the short-term nominal interest rate as an additional variable in the information set $\mathbf{Z}_t$, but the results hardly change. Given that the baseline forecasting VAR contains some insignificant coefficients, we check for the robustness of these results by restricting the lag order to $q = 3$ in model V. The resulting picture also suggests an unreliable estimate of the ratio of standard deviations accompanied by a sufficently precise estimate of the correlation coefficient. In a final specification, the variables are included in (demeaned) levels rather than in deviations from a linear trend. Here the results are alarming. The model is consistent with a substantially negative correlation between actual and fundamental inflation as well as with an almost perfect positive correlation.

The estimated slope coefficient γ in conjunction with a fixed discount factor β allows us to infer the average duration of sticky-price contracts under the Calvo price setting scheme. In the baseline specification we obtain an estimate of γ of 0.041, which implies a duration of fixed prices of 5.57 quarters. All other specifications suggest a duration of sticky-price contracts between five and eight quarters. Hence, these estimates are perfectly in line with existing macro evidence. Recently, extensive research on price stickiness in EMU countries based on micro data was carried out under the auspices of the Eurosystem's Inflation Persistence Network. The micro evidence points to a somewhat shorter duration of contracts with average durations of price spells ranging from four to five quarters.[4]

5 Conclusions

The standard New Keynesian Phillips Curve specifies current inflation as the present-value of the future stream of real marginal costs. Previous contributions to the literature exploited VAR projections of future real marginal costs to proxy market expectations and to derive a series of model-consistent or fundamental inflation rates. It is frequently argued that this series of fundamental inflation explains actual inflation quite well. In this paper we shed light on this finding using data for the Euro area. In particular, we used bootstrapped confidence bands to quantify the degree of estimation uncertainty around these estimates. We showed that the result for the purely forward-looking model cannot be interpreted as it is done in the literature due to the

[4] See the summary paper of Dhyne et al. (2004).

immensely wide confidence intervals.

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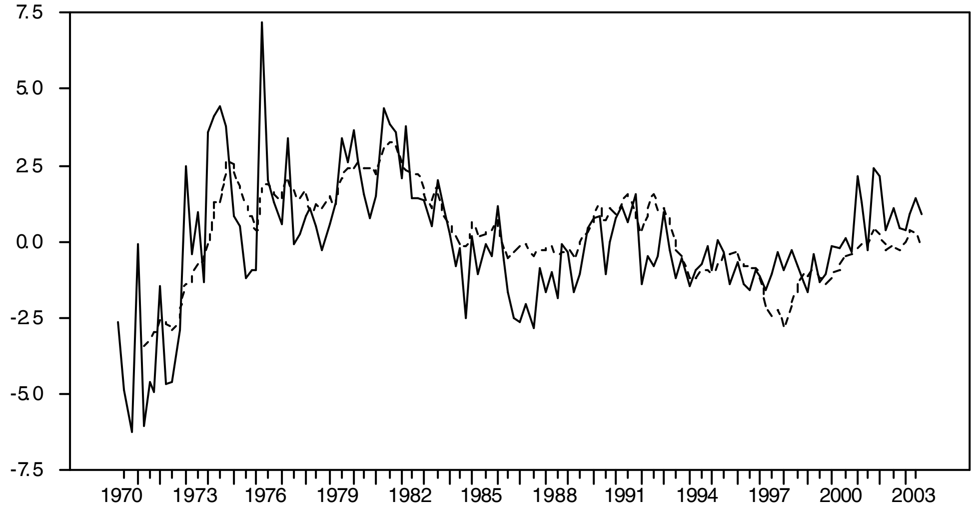

Figure 1: Actual (bold line) and fundamental (dotted line) inflation (detrended) in the Euro area (in % p.a.), derived from baseline model

Table 1: The fit of the forward-looking NKPC

	specification		results			
	VAR model	ϕ	measure of fit	estimate	90% band	*Duration*
I	$[rmc_t, \pi_t]'$	0.99	*std.dev.ratio*	0.70	$\left[0.34 \quad 1.82 \right]$	5.57
	detrended					
	$q = 5$		*corr.*	0.74	$\left[0.57 \quad 0.83 \right]$	
II	$[rmc_t, \pi_t]'$	0.98	*std.dev.ratio*	0.69	$\left[0.36 \quad 1.54 \right]$	5.53
	detrended					
	$q = 5$		*corr.*	0.73	$\left[0.57 \quad 0.82 \right]$	
III	$[rmc_t, \Delta nmc_t]'$	0.99	*std.dev.ratio*	0.54	$\left[0.20 \quad 1.14 \right]$	7.88
	detrended					
	$q = 5$		*corr.*	0.57	$\left[0.33 \quad 0.60 \right]$	
IV	$[rmc_t, \pi_t, i_t]'$	0.99	*std.dev.ratio*	0.55	$\left[0.15 \quad 1.87 \right]$	8.48
	detrended					
	$q = 3$		*corr.*	0.56	$\left[0.11 \quad 0.65 \right]$	
V	$[rmc_t, \pi_t]'$	0.99	*std.dev.ratio*	0.68	$\left[0.29 \quad 1.34 \right]$	6.23
	detrended					
	$q = 3$		*corr.*	0.70	$\left[0.59 \quad 0.77 \right]$	
VI	$[rmc_t, \pi_t]'$	0.99	*std.dev.ratio*	0.89	$\left[0.14 \quad 1.51 \right]$	10.63
	in levels					
	$q = 5$		*corr.*	0.88	$\left[-0.31 \quad 0.96 \right]$	