Debt limits and sovereign default in the euro area

Niccolò Battistini [*] [†]

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Abstract

This paper investigates fiscal sustainability through the lens of a dynamic general equilibrium model with strategic sovereign default. The model assesses the effects of macroeconomic fundamentals and fiscal policy on a government's default incentives and associated debt limit distributions. The standard Eaton-Gersovitz model is enriched with two features via novel applications to reflect salient features of euro area economies. First, the presence of domestic debt implies that a sovereign default, through lower repayments on sovereign debt, determines both benefits for the public sector and costs for private domestic investors. As the sovereign seeks to maximize domestic welfare, a higher domestic share of debt increases the government's incentives to honor its obligations and, thus, expands its borrowing opportunities. Second, the introduction of credible supranational fiscal rules creates the possibility for self-fulfilling debt crises and, accordingly, increases the borrowing costs for the government. In this way, fiscal rules reduce the sovereign's optimal level of debt and foster market-based fiscal discipline. In an empirical application, given the current state of the economy, the calibrated average euro area country faces a high risk of default, but its optimal debt and default policies imply that it can reap both welfare and sustainability gains through fiscal consolidation.

Keywords: debt limits, fiscal space, endogenous sovereign default, fiscal policy, fiscal rules, domestic debt, long-term debt, risk averse investors, euro area sovereign debt crisis
JEL Classification: E21, E43, F34, F44

[*]European Central Bank and Rutgers University. E-mail: niccolo.battistini@ecb.europa.eu.

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1 Introduction

Since the beginning of the recent sovereign debt crisis, euro area economies have confronted exceptional fiscal challenges. Dramatic recessions have exerted considerable pressures on euro area public finances across the board: large contractions in real output and consumption, coupled with rising unemployment rates, have led to persistent fiscal deficits, soaring sovereign debt levels as well as widening cross-country government bond differentials, as shown in Figure 1.[1] In this context, the debate on the measurement and assessment of the fiscal space available to euro area governments has gained ever more prominence in policy circles.[2] The notion of fiscal space crucially hinges on the dual objective of fiscal policy, namely ensuring macroeconomic stability and, *at the same time*, preserving debt sustainability.[3] The accurate identification of fiscal space is then essential to evaluating the ability of fiscal policy to stabilize macroeconomic fluctuations without endangering the sustainability of public finances.

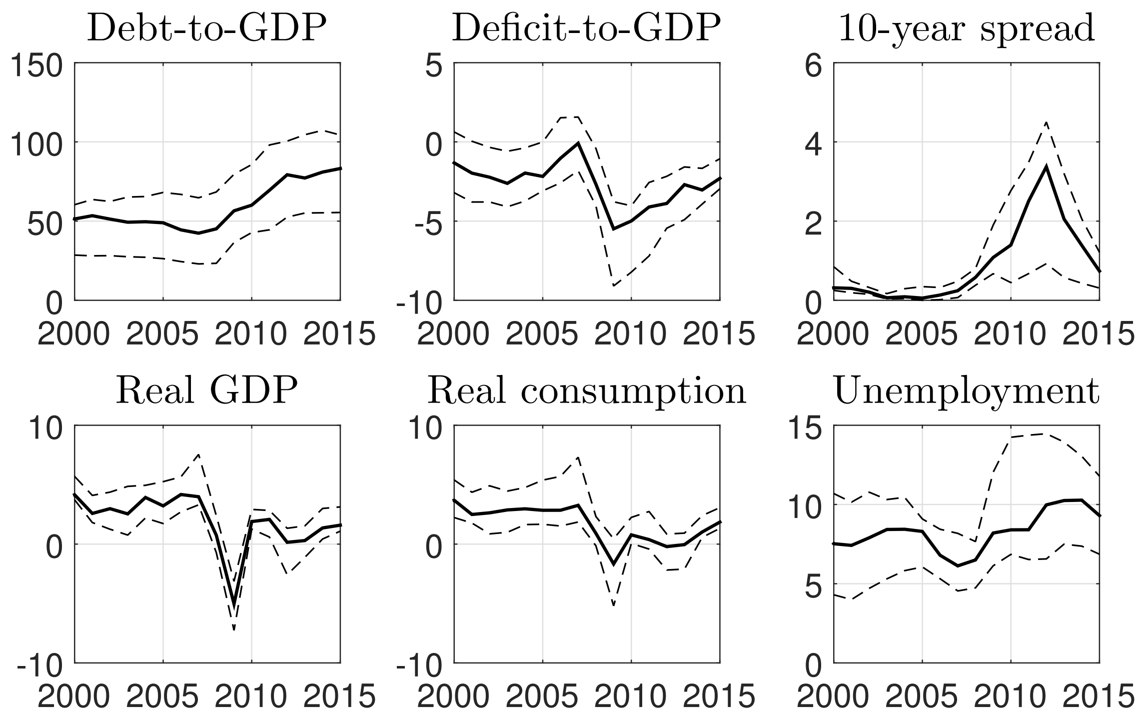
This paper contributes to the current debate by measuring and assessing fiscal space through the lenses of a dynamic stochastic general equilibrium model with strategic sovereign default à la Eaton and Gersovitz (1981). This paper puts forward a probabilistic, forward-looking notion of fiscal space, thus addressing the concerns of policy makers seeking to evaluate debt sustainability at different time horizons. More specifically, fiscal space is defined as the distance of the current debt level from its state-contingent debt limit, which is in turn considered as the maximum level of borrowing that the government is willing to honor within a certain time horizon. In this light, the model studies the implications of macroeconomic fundamentals and fiscal policy on the sovereign's incentives to default and the associated distributions of debt limits and fiscal space. Compared to the standard Eaton-Gersovitz framework, the model is endowed with several features aimed at reflecting important structural characteristics of euro area economies, such as risk averse investors, long-duration bonds, recovery rates over defaulted debt, taxes on consumption and labor income as well as, via novel applications, domestic debt and supranational fiscal rules.

[1] The median euro area country experienced its largest contraction in real GDP and real private consumption in 2009; at the same time, the median unemployment rate soared as the crisis unfolded up to 2015 without ever returning to its pre-crisis levels. Between 2007 and 2015, the receding real economy took its toll on fiscal balances, whereby the median debt-to-GDP ratio almost doubled and an initial median balanced budget was followed by repeated deficits. Concomitantly, a dramatic increase in the dispersion of cross-country sovereign spreads signaled a shift in market sentiments, as investors perceived bonds issued by different governments as imperfect substitutes and thus diversified between core and highly-indebted peripheral countries. For an encompassing description of the early stages of the euro area sovereign debt crisis, see Lane (2012).

[2] In the flagship document of European Union (EU) institutions, also known as the "Five Presidents' Report", Juncker et al. (2015) deem the presence of fiscal space as a necessary condition for the achievement of the optimal level of economic stabilization. Further, they recognize unsustainable fiscal policies as a major obstacle in the creation of a European fiscal governance framework and, generally, a full-fledged Economic and Monetary Union (EMU).

[3] In general, all definitions refer to a government's room of manoeuvre to provide resources for a desired purpose without endangering the sustainability of its financial position or the stability of the economy. The International Monetary Fund (2010) succinctly defines fiscal space as "the scope for further increases in public debt without undermining sustainability".

Figure 1: Public finances and real economy in the euro area between 2000 and 2015

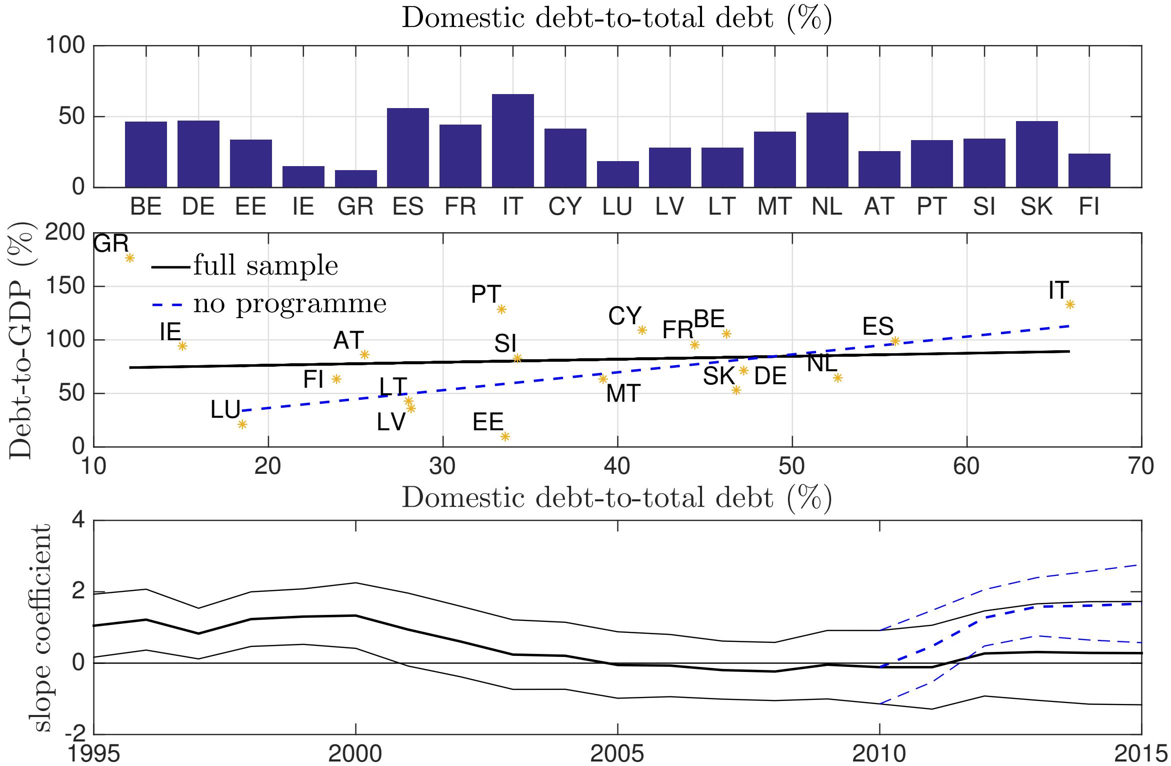


Sources: Eurosystem.
Notes: Proceeding leftwards and downwards, the charts show the debt-to-GDP ratio, the deficit-to-GDP ratio, the 10-year government bond spread (relative to the Bund yield), the real GDP growth, the real private consumption growth, the unemployment rate in 19 euro area countries. Each chart shows the median (solid line) and the interquartile range (area between the dashed lines) of the cross-sectional distribution. All variables are reported in percentage terms.

As regards the first novel feature of the model, domestic debt is introduced via a two-stage process. In the first stage, domestic investors interact with foreign lenders to bargain the optimal domestic share of total debt on primary markets and post their bids in a binding auction with the government. In the second stage, trades between lenders and the government take place on perfectly competitive secondary markets, given the overall price for debt.

The inclusion of domestic debt is essential to investigating sovereign risk and its implications for debt sustainability and welfare in advanced economies and, especially, in the euro area. The importance of domestic debt stems from four empirical regularities. First, domestic debt constitutes a considerable fraction of government debt. In a panel of 43 emerging and advanced economies in 2011, the median domestic share of total debt exceeds 50% (Mallucci, 2015), whereas the same figure for euro area countries in 2015 is a still sizeable 34% (with a peak of 66% for Italy), as shown in the first chart of Figure 2. Second, domestic default occurs frequently, implying that, against conventional wisdom, domestic lenders are not senior to external lenders: in a panel of 30 developing countries, domestic default occurs in

Figure 2: Debt ratio and domestic share of debt



Sources: Eurosystem.
Notes: The first chart shows the share of total government debt held by domestic residents in 19 euro area countries. In the second chart, the solid (black) line is the regression line obtained from regressing the debt-to-GDP ratio on the domestic-to-total debt ratio for the full sample of 19 euro area countries at the end of 2015; the dashed (blue) line refers to the same regression for the full sample except the (ex-)programme countries (i.e. Greece, Ireland, Portugal and Cyprus). In the third chart, the same regression is repeated for every year from 1995 to 2015 and the same line convention is used; further, the thick lines represent the point estimates, whereas the thin lines show the 95% confidence intervals.

48% of crises episodes between 1980 and 2005, whereas the figure increases to 73% between 1990 and 2005 (Mallucci, 2015; see also Reinhart and Rogoff, 2011). Third, mainly through a reduction in external and domestic credit to the domestic non-financial private sector, output contractions are significantly more pronounced when the sovereign reneges debt contracts that are also held domestically, as opposed to defaults on debt exclusively held by foreign creditors (Arteta and Hale, 2008, Sandleris, 2015, and Gennaioli, Martin and Rossi, 2014; for the case of a euro area country, see Albertazzi et al., 2014). Fourth, in euro area economies, higher domestic holdings of sovereign debt are associated with higher debt ratios. As shown in the second chart of Figure 2, regressing the debt-to-GDP ratio on the domestic share of debt for a cross-section of euro area countries in 2015 yields a positive slope coefficient of 0.3, which is not statistically significant at the 10% significance level. Yet, this result may stem from the fact that macroeconomic adjustment programmes have biased the correlation between domestic and total debt, as supranational or bilateral cross-country financial assistance considerably reduced the share of debt held domestically with negligible effects on the amount of total debt. When the same regression is run excluding (ex-)programme countries (i.e. Greece, Ireland, Portugal and Cyprus), the slope coefficient increases to 1.7 (thus, a more than one-to-one relationship) and becomes significant. The third chart of Figure 2 depicts the series of slope coefficients obtained through the same regression repeated each year from 1995 to 2015 with a full cross-sectional sample (solid line) and excluding programme countries as of the beginning of their financial assistance (dashed line). Clearly, between the inception of the EMU and the start of the crisis, increases in domestic debt were not accompanied by increases in total debt. However, the positive relationship between domestic and total debt emerges before the creation of the EMU (for the full sample) and after the start of the crisis (excluding programme countries). Ultimately, this evidence hints at the presence of incentives towards debt accumulation in countries featuring large shares of total debt held by domestic residents, as long as no distortions affect the pricing of country-specific default risk.

As regards the second novel feature of the model, the government is assumed to interact with supranational fiscal authorities. The latter require compliance with fiscal rules, but, in the absence of any enforcement power and commitment technology, the government may optimally choose to deviate. If the government faces tight borrowing constraints and deviates from fiscal rules, lenders receive a negative signal about its credibility and trigger a sudden stop in capital flows. Given the possibility of self-fulfilling roll-over crises via a credibility channel, fiscal rules reduce the sovereign's incentives to borrow, hence decreasing its optimal debt levels and fostering market-based fiscal discipline.

The inclusion of fiscal rules is motivated by their increased importance as means to foster fiscal discipline in recent years. Over the last decades, an increasing number of countries has relied on fiscal rules to guide policy.[4] In the European context, the recent sovereign debt crisis paved the way for the inclusion of fiscal rules within the EU fiscal governance framework as well as their anchoring in national legislation.[5] Notwithstanding difficulties in evaluating the desirability of fiscal rules (see, for instance, Afonso and Hauptmeier, 2009), several studies investigate their impact on a government's fiscal space. In their empirical investigation of national fiscal governance frameworks in the EU, Nerlich and Reuter (2015) show that national fiscal rules contribute to increasing the available fiscal space while reducing the degree of procyclicality of national fiscal policies. In an analysis of European bond spreads before the financial crisis, Heinemann, Osterloh and Kalb (2014) find that the interaction of stability preferences and fiscal rules points to a particular potential for the latter to restore market

[4]According to the International Monetary Fund (2009), 80 countries had national and/or supranational rules in place as of 2009 compared to only 7 countries in 1990. According to the database on numerical fiscal rules prepared by European Commission, 25 EU countries had some kind of fiscal rule in place in 2014, up from only 6 countries in 1990.

[5]The so-called "Two-Pack" regulation required Member States to transpose the fiscal rules envisaged under the Stability and Growth Pact into national legislation, as well as the set-up of independent national bodies monitoring compliance with the fiscal rules.

confidence in countries with a historical lack of stability culture.[6]

As regard the results under the benchmark calibration, a sensitivity analysis shows the effects of different structural characteristics (as represented by specific parameter values) on the sovereign's incentives to default and the associated debt limit distributions. First, a higher domestic share of total debt entails higher default costs, due to the losses incurred by domestic investors, thus increasing the sovereign's incentives to repay and expanding its borrowing opportunities. Second, a higher risk aversion for the private sector implies that domestic investors value relatively more the benefits of consumption smoothing; then, default is relatively more costly, and the sovereign has lower incentives to renege its obligations and faces higher debt limit distributions. Third, when the volatility of productivity shocks increases, the debt limit distribution becomes more disperse and borrowing opportunities shrink as the economy may be subject to deeper recessions. Fourth, as the costs of default for the overall economy increase, either by decreasing the probability of receiving a settlement offer or increasing the exogenous productivity loss, the sovereign has higher incentives to honor its obligations. Finally, as the steady-state marginal utility of the private (relative to the public) sector increases, for instance through higher tax rates or lower public-to-private consumption ratios, the sovereign finds default less attractive and hence faces a looser borrowing constraint.

As regards the analysis of fiscal space in light of the dual objective of fiscal policy, macroeconomic stabilization and debt sustainability are assessed under different configurations of the model. First, the presence of fiscal rules does not worsen sustainability concerns as it does not affect the government's fiscal space. Yet, the possibility of self-fulfilling roll-over crises fosters market-based fiscal discipline by reducing the sovereign's optimal level of debt. In addition, the average euro area country is not subject to the conventional trade-off between macroeconomic stabilization and debt sustainability. On the one hand, the benchmark economy faces a high risk of default, since at the current debt level, the associated default probability exceeds the maximum threshold associated with an investment-grade government bond. On the other hand, the domestic government finds it optimal to reduce its borrowing level in order to maximize the overall welfare for the economy. Hence, in the particular case of the benchmark calibration, the average euro area country can simultaneously reap welfare and sustainability gains through a fiscal consolidation.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents and analyzes the theoretical features of the baseline model. Section 4 extends the baseline model to allow for the presence of a government (1) without commitment to fiscal rules and (2) maximizing its tax receipts. Section 5 contains the quantitative assessment, including the numerical solution, sensitivity analysis and empirical application of the model for the average euro area country. Section 6 puts forward concluding remarks.

[6]Further theoretical and empirical evidence on the effects of fiscal rules in the context of European Union countries, see Pappa and Vassilatos (2007) and Plödt and Reicher (2014).

2 Related literature

This paper is at the cross-way between two main strands in literature extant on sovereign debt. First, as regards the literature on fiscal sustainability, a number of studies has advanced different measures of fiscal space. The methodological approaches to quantify fiscal space may be grouped in three broad categories. First, rules-based fiscal frameworks measure fiscal space as the distance of a budgetary indicator from a medium-term objective (European Commission, 2016). Second, a comprehensive framework for debt sustainability analysis (DSA) evaluates the overall sustainability of public finances through a large set of significant indicators (e.g. DSA toolboxes developed at several international organizations). Third, model-based approaches typically gauge fiscal space as the distance of the debt-to-GDP ratio from the debt limit, defined as the maximum amount of debt a government is able or willing to honor (e.g. International Monetary Fund, 2010, and Moody's, 2016). Model-based frameworks may analyze a government's debt limit via either a reduced-form (e.g. Ghosh et al., 2013 and Collard, Habib and Rochet, 2015) or a structural-form approach, depending on whether the sovereign's option to default is explicitly modeled. Among structural frameworks, a class of models conceives default as a random event linked to the government's *ability* to service its debt (e.g. Davig, Leeper and Walker, 2011, Bi, 2012 and Polito and Wickens, 2014);[7] a different class of models posits default as a strategic event based on the government's *willingness* to honor its obligations (e.g. Aguiar and Gopinath, 2006, and Arellano, 2008).[8]

Although the implementation of policies aimed at deciding the optimal use of fiscal space ultimately depends on political and institutional factors, methodological approaches for fiscal sustainability may effectively guide policy. For instance, they may anchor decisions to consistent targets (rules-based frameworks) or show the effects of diverse scenarios on the future path of the debt level (DSA-based frameworks and reduced-form model-based frameworks). In this light, structural model-based frameworks are particularly suitable for policy evaluation, as forward-looking agents define their optimal policies based on clearly discernible assumptions, thus being immune from the Lucas (1976) critique.[9] Finally, structural model-based frameworks related to the government's willingness to repay involve aggregate welfare considerations. Hence, they may inform the policy debate on fiscal space by assessing not only the feasibility of specific fiscal policies, but also about their desirability.

[7] In these models, sovereign default is ultimately the result of a "divine coincidence", or *bad luck*, occurring when the debt ratio is above a threshold randomly drawn from the debt limit distribution, constructed as the discounted sum of expected budgetary outcomes. Polito and Wickens (2014) describe alternative approaches (see also Sachs, 1989, and Aiyagari, 1994).

[8] In this class of models, sovereign default occurs as an outcome of a strategic decision by the government weighing the utility of repaying against the utility of reneging its debt. The notion of debt limit hence derived in the framework studied by Eaton and Gersovitz (1981) retains similarities with the concepts studied, for instance, by Lucas (1985) and Zhang (1997).

[9] However, prudence requires distinguishing debt limits from optimal debt levels, whereby the latter are typically well below the former (see, for instance, the discussions in Davig, Leeper and Walker, 2011, and Ghosh et al., 2013).

As this paper lays out a model based on the latter class of models, it is related to the literature on unsecured sovereign debt and default. In the baseline version of quantitative models with endogenous sovereign default, pioneered by Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008), a small open economy can access incomplete international financial markets where only state non-contingent assets are available. In the absence of any commitment technology, the government determines its default policy on outstanding debt on the basis of the future discounted values of repayment and default. Default may endogenously occur in equilibrium depending on the trade-off between the benefits from debt relief and the costs from temporary financial autarky and loss in output. In turn, international lenders charge risk premia on sovereign borrowers, thus pricing in the probability of default. A positive feature of this class of models is their fair ability to replicate several dynamics and moments of macroeconomic variables, notably in emerging countries.[10]

The contribution of this paper to the second strand of literature is threefold. First, as regards domestic debt, this paper mainly relates to Mallucci (2015) and Engler and Große Steffen (2016), who introduce domestic debt as part of domestic intermediaries' asset holdings to explain the contraction of credit determined by a sovereign default. Both studies need two different state variables for domestic debt and total or foreign debt linked to domestic intermediaries' net worth. The comparative advantage of this paper stems from the assumption that domestic debt is held by domestic agents for simple consumption smoothing purposes, so that domestic investors' net worth is implicitly represented by output. So, the model includes domestic debt in a parsimonious way, as no additional state variable is required.

Second, this paper introduces fiscal rules imposed by supranational fiscal authorities. In this regard, this model relates to the works by Hatchondo, Martinez and Roch (2015) and Arellano and Bai (2016). Similarly to both studies, this paper measures the desirability of fiscal rules via welfare gains. In contrast to them, this model does not assume the presence of a perfect commitment technology that enforces compliance of budgetary policies with fiscal rules. Conversely, the government is allowed to deviate from fiscal rules and thus, conditional on facing a binding borrowing constraint, trigger self-fulfilling roll-over crises.

Third, this paper advances a novel measure of debt limit and proposes economically meaningful thresholds relevant for policy makers seeking to address sustainability concerns in the short-to-medium term. In particular, this paper advances a forward-looking, probabilistic notion of debt limit contingent on the current state of the economy.[11]. Further, three thresholds are put forward to pin down meaningful points in the spectrum of sovereign borrowing opportunities, thus identifying debt limits at different levels of riskiness and time horizons.

[10] See, for instance, Neumeyer and Perri (2005).

[11] In contrast, deterministic debt limits in the context of sovereign default may be found in Aguiar and Gopinath (2006), on the basis of computations à la Lucas (1985), and Arellano (2008), on the basis of considerations in Zhang (1997)

3 The model

In a small open economy, households, firms and the government interact with a pool of domestic and foreign lenders. Households have preferences defined over private and public consumption and labor. They consume, work and are entitled to the entire profits of firms and domestic lenders. Firms produce consumption goods combining labor and total factor productivity (TFP) via a production function with constant returns to scale. The government is benevolent and seeks to insure households' welfare against TFP volatility. It finances public expenditures by borrowing from abroad and taxing households' consumption and labor income. Financial markets are incomplete, as the only traded assets are non-contingent long-term bonds that mature probabilistically. Debt contracts are not enforceable, since the government has the option to default on their total amount. When the government repudiates its outstanding debt, the economy is temporarily in financial autarky and incurs an exogenous TFP loss. Domestic and foreign lenders bargain the optimal domestic share of total sovereign debt on primary markets and post their bids in a binding auction with the government, which in turn determines the issuance of total debt given the overall price prevailing in secondary markets.

3.1 Households

Time is discrete and denoted as $t \in \{0, 1, 2, ..., \infty\}$. Infinitely-lived identical households feature the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, g_t), \tag{1}$$

where E_0 denotes the expectations operator at time zero, β households' subjective discount factor, $u(c_t, h_t, g_t)$ the strictly increasing, concave and twice differentiable instantaneous utility function, c_t private consumption, h_t hours worked and g_t public consumption at time t. Households take other agents' choices as given and determine their optimal level of private consumption and working hours subject to the flow budget constraint below:

$$(1 + \tau_t^c)c_t = (1 - \tau_t^h)w_t h_t + p_t^E + p_t^H, \tag{2}$$

where τ_t^c and τ_t^h indicate the tax rates on consumption and labor income, respectively, while w_t per-hour wages and p_t^E and p_t^H profits received from firms and domestic lenders owned by domestic households, respectively, in units of consumption goods. The first-order conditions with respect to c_t and h_t then determine the solution for the households' problem:

$$-\frac{u_c(c_t, h_t, g_t)}{u_h(c_t, h_t, g_t)} = \frac{1 + \tau_t^c}{(1 - \tau_t^h)w_t}, \tag{3}$$

where $u_x(c_t, h_t, g_t)$ is defined as the derivative of $u(c_t, h_t, g_t)$ with respect to variable x_t. Equation (3) determines the equilibrium labor supply function.

3.2 Firms

Firms seek to maximize their profits, which are given by:

$$p_t^E = y_t - w_t h_t, \tag{4}$$

where output y_t is produced by combining labor services h_t and TFP a_t via a production function $f(h_t)$ with constant returns to scale:

$$y_t = a_t f(h_t). \tag{5}$$

The stochastic process for a_t is assumed to follow a first-order Markov sequence with transition probability function $F(a_t|a_{t-1})$, which is defined over the finite set $A = \{a^1, ..., a^N\} \subset \mathbb{R}_{++}$ and approximates a first-order autoregressive (AR) process:

$$\log a_t = (1 - \rho_a) \log \bar{a} + \rho_a \log a_{t-1} + \varepsilon_t^a, \tag{6}$$

with ρ_a and $\bar{a}$ denoting the AR coefficient and the long-run mean of the TFP process, respectively, $\varepsilon_t^a \sim N(0, \sigma_a^2)$ TFP shocks and σ_a their standard deviation. The first-order condition with respect to h_t solves the firms' problem and yields the optimal condition below:

$$w_t = a_t f_h(h_t), \tag{7}$$

which defines the equilibrium labor demand function.

3.3 The government

The sovereign government is benevolent and its objective is to maximize households' welfare. The sovereign determines its optimal policies by deciding (1) whether to repay or default on the total amount of its outstanding debt, (2) its fiscal policy, including public expenditures and taxes on consumption and labor income and, if repayment is optimal, (3) the amount of debt to issue or purchase on capital markets.

3.3.1 Debt and default policy

Each period, conditional on having access to credit markets, the government exerts its option to default d_t deciding whether to honor or default on the total amount of its outstanding debt

b_t by comparing the net benefits of the two options.[12]

On the one hand, if default is optimal ($d_t = 1$), the defaulting government suffers an exogenous cost in productivity $\omega(a_t)$.[13] In addition, the government foregoes the benefits of consumption smoothing due to the economy's exclusion from credit markets. However, financial autarky is temporary as the economy can return to international credit markets. Following Hatchondo, Martinez and Sosa Padilla (2014) and Hatchondo, Martinez and Roch (2015), as of the period after default, the government has the opportunity to end the default with a constant probability θ. In this case, the government may end default by exchanging delinquent debt with bonds promising to pay $1-\delta \in (0,1)$ times the payments promised by the exchanged debt. If the government rejects the deal and continues in default, its debt level remains $1-\delta$ times the debt level before the restructuring opportunity. Hence, the recovery rate gradually declines with the time spent by the country in financial autarky. However, during default, the government's payment obligations grow at the risk-free interest rate r. Notice that δ may be interpreted as the government's haircut, or actual default rate, on the net present value of its total liabilities;[14] further, the expected duration of financial autarky without restructuring opportunities is $1/\theta$.

On the other hand, if repayment is optimal ($d_t = 0$), the government weighs the costs of lowering public consumption to repay the non-contingent loan against the benefits of increasing private consumption by servicing its debt. Further, the government retains the option to borrow or lend in international credit markets by selling or buying bonds, respectively. Financial markets are incomplete, since debt contracts are state non-contingent claims to future units of consumption goods. Long-term bonds are assumed to mature probabilistically, as in Chatterjee and Eyigungor (2012). Each unit of debt matures next period with probability λ and gives out a coupon payment z with probability $1-\lambda$. Thus, the expected duration of bonds can be computed as $1/\lambda$. Let the total amount of bonds issued at t be denoted by b_{t+1}. Since unit bonds are assumed to be infinitesimally small, the issuer's coupon and principal obligations next period will be $z(1-\lambda)b_{t+1}$ and λb_{t+1}, respectively, with certainty.[15]

[12] As noted in previous studies with long-duration bonds (see, for instance, Hatchondo and Martinez, 2009, Chatterjee and Eyigungor, 2012, and Arellano and Ramanarayanan, 2012), the assumption of repudiation of all current and future debt obligations is consistent with the actual behavior of defaulting governments. Sovereign debt contracts often contain two types of clauses. The acceleration clause allows all creditors to call their debt in case the government defaults on a payment. The cross-default clause implies that, after a default event, future debt obligations become current, since a default on any government obligation also constitutes a default on the contracts containing that clause.

[13] Following Chatterjee and Eyigungor (2012), the loss function is assumed to be quadratic in TFP, i.e. $\omega(a_t) = \min\{a_t, \max\{0, \omega_1 a_t + \omega_2 a_t^2\}\}$ so that $a_t - \omega(a_t)$ is greater than zero. Moreover, $\omega_2 < \frac{1-\omega_1}{2a^N}$ so that $a_t - \omega(a_t)$ is strictly increasing in a_t for any $a_t \in A = \{a^1, ..., a^N\}$. Finally, $\omega_1 < 0$ and $\omega_2 > 0$ so that the cost is nil for $0 \leq a_t \leq -\frac{\omega_1}{\omega_2}$ and rises more than proportionately with TFP for $a_t > -\frac{\omega_1}{\omega_2}$, resembling the asymmetric loss function in Arellano (2008). Mendoza and Yue (2012) show that this type of loss function may arise endogenously due to the reduction in international trade in the aftermath of a sovereign default.

[14] In this way, the model introduces a concept of recovery rate of debt in default similar to those studied by, for instance, Sturzenegger and Zettelmeyer (2008) and Cruces and Trebesch (2013).

[15] Notice that unit bonds of type (z, λ) issued at different periods in the past have exactly the same payoff

Every period the government can choose its optimal debt level b_{t+1} for the following period, anticipating that the price q_t for selling or purchasing bonds is such that lenders make zero profits in expectation. As only λb_t debt matures every period, the government can repurchase non-maturing bonds $(1-\lambda)b_t$ and issue new bonds b_{t+1} at the zero expected-profit price on secondary markets.

3.3.2 Fiscal policy

Following its optimal default and debt decisions, the sovereign determines its fiscal policy. Public spending can be financed through foreign debt and domestic taxes on households' consumption and labor income. As shown by Cuadra, Sanchez and Sapriza (2010), in a model with unenforceable sovereign debt contracts, optimal taxation engenders a pro-cyclical fiscal policy. Since the repayment of non-contingent loans is more costly in recessions, the incentives to default are higher in bad times. Thus, in recessions the government faces higher opportunity cost of borrowing due to higher default risk and finds it optimal to rely more heavily on taxation to finance public expenditures. Conversely, in expansions cheaper credit determines an increase in financing through borrowing, while taxes play a lesser role. Thus, tax rates are pro-cyclical, since lower (higher) tax rates occur during good (bad) times.

However, several empirical studies observe how fiscal policies vary across countries featuring heterogeneous economic and institutional characteristics. For instance, Talvi and Végh (2000) observe a-cyclical and pro-cyclical fiscal policies in G7 and developing economies, respectively, and explain the difference as due to political failures typical of emerging countries in insuring welfare against volatile macroeconomic fluctuations. Further, Kaminsky, Reinhart and Végh (2004) associate similar patterns with capital flow cycles, exchange rate regimes and financial market integration. Therefore, a quantitative model seeking to replicate the features of actual fiscal policies across countries with different degrees of economic and institutional development as well as their implications for debt sustainability and welfare should allow for a flexible specification.

Hence, in the baseline model, the government is assumed to tax consumption and labor income according to the targeting rules below:

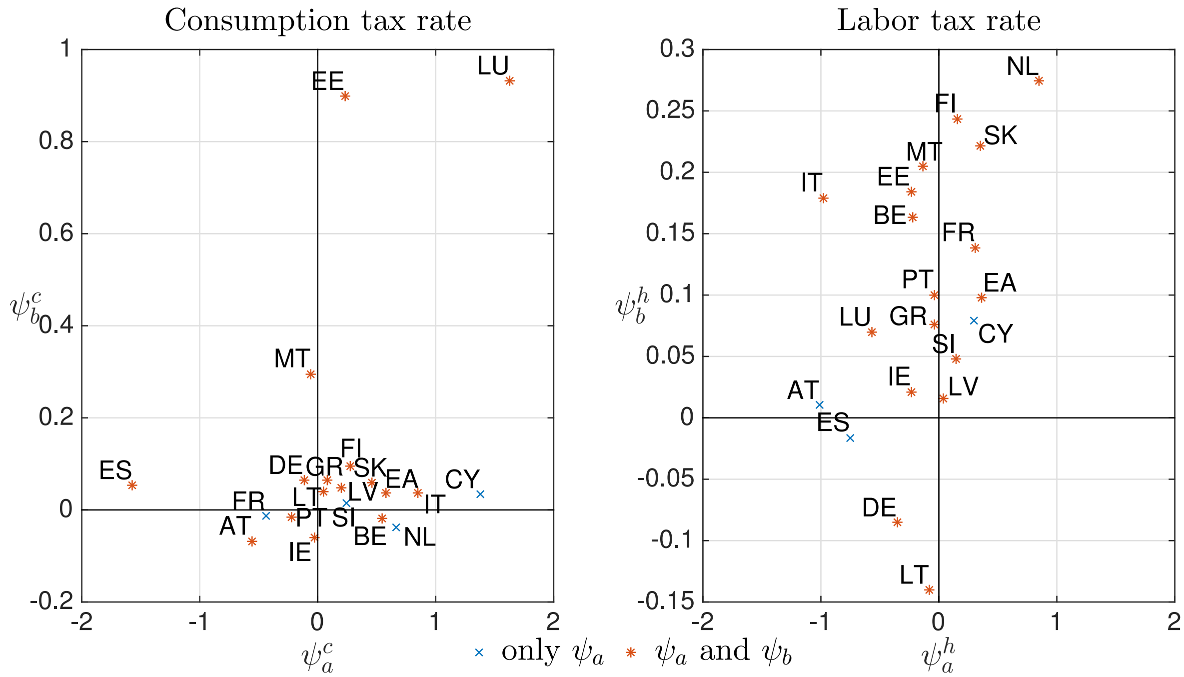
$$\tau_t^c = \bar{\tau}^c + \psi_a^c(a_t - \bar{a}) + (1-d_t)\psi_b^c(b_t - \bar{b}) \tag{8a}$$

$$\tau_t^h = \bar{\tau}^h + \psi_a^h(a_t - \bar{a}) + (1-d_t)\psi_b^h(b_t - \bar{b}), \tag{8b}$$

where τ_t^c and τ_t^h refer to the tax rates on consumption and labor income, $\bar{\tau}^c$ and $\bar{\tau}^h$ denote the tax rate on consumption, the tax rate on labor income and the amount of debt, respectively,

structure. This formulation implies that bonds are "memoryless", whereby one needs to keep track of the total number of bonds only, thus limiting the number of state variables. Alternative formulations of long-term debt in the literature extant have used assumptions with similar implications (see, for instance, Hatchondo and Martinez, 2009 and Arellano and Ramanarayanan, 2012).

Figure 3: Estimates of debt and productivity elasticities of tax rates



Sources: Eurostat, Eurosystem.
Notes: Markers for each point in the scatter plot refer to the statistical significance of the parameters as identified in the legend, on the basis of two-tail tests at the 10% significance level.

in the risk-free (default-free) deterministic steady state of the model economy, and $\bar{b}$ is an economic or institutional constant critical level for debt.[16] Further, ψ_j^l indicates the elasticity of the tax rate on variable $l \in \{c, h\}$ with respect to the deviation of variable $j \in \{a, b\}$ from its constant critical level. Figure 3 plots the estimates of the elasticities ψ_j^l of tax rates with respect to TFP and debt in all euro area countries according to Equations (8).[17] Notice that a significantly positive (negative) ψ_a^l for $l \in \{c, h\}$ may be interpreted as representing a counter-cyclical (pro-cyclical) tax policy, while a significantly positive (negative) ψ_b^l for $l \in \{c, h\}$ may be considered as associated with a generally sustainable (unsustainable) tax policy.[18] In line

[16]Details on the identification of the critical level $\bar{b}$ are provided in Section 5.1 below.

[17]Details on the estimation sample and the model specification can be found in Section 5.1 below.

[18]Leeper and Leith (2016) show that in a simple model, where the government follows a debt-targeting surplus rule, a non-explosive equilibrium exists as long as the primary surplus has a sufficiently positive response to increases in real debt. When this condition is not satisfied, solutions with unbounded debt inevitably rely on non-distorting taxes, which permit revenues to grow forever at the same rate as interest receipts on government bond holdings. However, in this model, a negative (or not sufficiently positive) ψ_b^l need not lead to unbounded equilibria for two main reasons. First, unsustainable tax rates on either tax base may be more than compensated by sustainable tax rates on the other tax base. Second, the pricing of default risk entails higher costs and lower borrowing opportunities for the government when its outstanding debt is high; thus, the possibility of default naturally eliminates the possibility of unbounded debt in equilibrium. In the rare event of an unbounded solution, a negative ψ_b^l for $l \in \{c, h\}$ is set to zero to ensure convergence of the bond

with aforementioned empirical evidence, a diverse picture emerges from the estimation of the responses of governments to economic fluctuations and debt levels across the euro area. For instance, economic and institutional differences may explain pro-cyclical and counter-cyclical indirect taxation in two often-thought similar countries such as Spain and Italy, respectively. Likewise, France and Germany, typically belonging to the host of euro area core countries, exhibit sustainable and unsustainable direct taxation patterns, respectively.

Finally, given τ_t^c and τ_t^h, the government's flow budget constraint determines public spending g_t as follows:

$$g_t = \tau_t^c c_t + \tau_t^h w_t h_t + (1 - d_t)\{q_t[b_{t+1} - (1 - \lambda)b_t] - [\lambda + (1 - \lambda)z]b_t\}. \tag{9}$$

As in Cuadra, Sanchez and Sapriza (2010), public spending provides direct utility to the private sector, which prefers a smooth path of public spending over a volatile one. Yet, as non-contingent bonds are not good instruments for consumption smoothing purposes, the government is not able to smooth public spending, so that public expenditures are pro-cyclical. Therefore, the government optimally implements a pro-cyclical expenditure policy.

3.3.3 Recursive formulation

The intertemporal problem of the government can be expressed in a recursive dynamic programming form. This model focuses on Markov perfect general equilibria: in each period, the government's equilibrium strategies depend only on payoff-relevant state variables.[19] The government makes all its decisions given the state of the economy, i.e. TFP a_t, the amount of outstanding foreign debt b_t and the credit situation of the country d_t. Conditional on having access to credit markets, the sovereign decides whether to honor its debt, then determining the amount of borrowing b_{t+1}, or default. In either case, after choosing its optimal default strategy, the government defines its fiscal policy, including the tax rates on consumption τ_t^c and labor income τ_t^h as well as the level of public expenditures g_t.

Conditional on a good credit standing, the government exerts its default option by solving the following problem:

$$V(a_t, b_t) = \max_{d_t}\{(1 - d_t)V^R(a_t, b_t) + d_t V^D(a_t)\}, \tag{10}$$

which denotes the general continuation value of the government before the determination of

price/value function iterations. In any case, tax policies are dubbed (un)sustainable for the sake of convention, although tempered by the term "generally".

[19] As discussed by Krusell and Smith (2003), infinite-horizon economies may feature multiple Markov perfect equilibria. In order to avoid this problem, the numerical solution of the model entails a backward procedure simulating a finite-horizon economy. As the number of periods increases until the bond price function and the value function for the first and second periods of this economy converge, the first-period equilibrium functions are considered equilibrium functions of the infinite-horizon-economy. The outline for the solution strategy is sketched in Section 5.2.

a default policy. If the government decides to honor its debt, then it determines the optimal borrowing level as follows:

$$V^R(a_t, b_t) = \max_{b_{t+1} \in B} \{u(c_t, h_t, g_t) + \beta E_t[V(a_{t+1}, b_{t+1})]\} \tag{11}$$

subject to the equilibrium conditions (2), (3), (4) and (7) for the private sector as well as (8) and (9) for fiscal policy. Notice that the borrowing decision is defined over a finite set $B = \{b^1, ..., b^M\}$, whereby an upper bound on debt, i.e. $b_{t+1} \leq b^M$, prevents the government from engaging in Ponzi schemes but does not bind in equilibrium. If the government reneges its debt, then it faces the following lifetime utility:

$$\begin{aligned} V^D(a_t, b_t) =& \, u(c_t, h_t, g_t) \\ &+ \beta E_t[(\theta V(a_{t+1}, (1-\delta)(1+r)b_t) + (1-\theta)V^D(a_{t+1}, (1+r)b_t)], \end{aligned} \tag{12}$$

where the same equilibrium conditions for the private sector and fiscal policy hold and TFP under default is determined as $a_t - \omega(a_t)$. The government's continuation value under default reflects the possibility to return to international credit markets with recovered debt the following period with probability θ. The presence of recovered debt (growing each period at the risk-free rate) implies that the value of default depends on outstanding debt as well as the current productivity level.

3.4 Lenders

A pool of domestic and foreign lenders interact on international capital markets with the domestic sovereign and have perfect information regarding the economy's realized TFP shocks. The pool of lenders can coordinate so that the government must satisfy the conditions of both groups of creditors in order to access domestic and international capital markets. The two groups of lenders interact with each other and with the sovereign in two stages.

3.4.1 First stage: Primary markets

In the first stage, primary markets open and an auction takes place where both domestic and foreign lenders post their respective bids on the share of total sovereign debt they are willing to purchase. At the same time, the two groups choose their optimal share of sovereign debt by engaging in a Nash bargaining game in which each counterpart's bargaining power is constant over time. Domestic lenders determine the domestic fraction b^H_{t+1} of total sovereign debt b_{t+1} issued by the government *vis-à-vis* foreign lenders by solving the following problem:

$$\max_{b^H_{t+1} \in (0,1)} \{(E_t p^H_{t+1})^{1-\zeta}(E_t p^F_{t+1})^{\zeta}\}, \tag{13}$$

subject to

$$E_t p^H_{t+1} \geq 0 \tag{14a}$$

$$E_t p^F_{t+1} \geq 0, \tag{14b}$$

where $\zeta \in [0, 1]$ denotes foreign lenders' bargaining power, and $E_t p^H_{t+1}$ and $E_t p^F_{t+1}$ are the expected profits of domestic and foreign lenders. Equations (14) ensure that each group in the pool of lenders is willing to participate in the bargaining game. Hence, lenders are willing to purchase their respective share of government bonds only if their expected profits satisfy the conditions of each individual type.

As customary in the literature, foreign lenders can trade riskless assets on international capital markets at the risk-free rate r and are assumed to be risk neutral. In contrast, domestic lenders can only trade domestic sovereign bonds. Since they are owned by domestic households, domestic lenders are assumed to be risk averse, thus charging a premium on the actuarially fair price. As argued, for instance, by Attinasi, Checherita-Westphal and Nickel (2009), Longstaff et al. (2011), Kennedy and Palerm (2014) and Cimadomo, Claeys and Poplawski-Ribeiro (2016), investor risk aversion is a major driver of the dynamics of government bond yields across emerging and advanced economies. Following Arellano and Ramanarayanan (2012), the pricing kernel is modeled as a function of the borrower's income.[20] Domestic lenders' bond price kernel is given by $M(a_{t+1}|a_t) \equiv \exp(-(1-\beta)/\beta - \sigma\varepsilon^a_{t+1} - \frac{1}{2}\sigma^2\sigma^2_a)$, where ε^a_{t+1} is defined by Equation (6). This definition of the stochastic discount factor is a special case of the discrete-time version of the one-factor model of the term structure proposed by Vasicek (1977) and Backus, Foresi and Telmer (1998). As households own domestic lenders, they share the same preferences, whereby the subjective discount rate is given by $(1-\beta)/\beta$ and the market price of risk by σ.[21] This specification implies that $M(a_{t+1}|a_t)$ is negatively correlated with domestic lenders' payoff next period and bond prices reflect compensation for the risk of a sovereign default in states when investors have high marginal utility.[22] The risk premium then comes from the interaction of the lenders' pricing kernel with default outcomes and future bond prices.

[20] The pricing kernel is a function of only the borrower's income because it is a parsimonious way to model risk premia that vary with the probability of default. This method has the advantage of avoiding (1) the introduction of an additional exogenous state variable into the model and (2) the computational burden of non-linear numerical solutions for each grid point of the state space and at each bond price/value function iteration. Although theoretically appealing, modeling the discount factor as the marginal rate of substitution between consumption today and tomorrow would inevitably forgo these benefits.

[21] In a similar environment, Lintner (1970) shows that in purely competitive markets in which investors with given constant risk aversion choose their investment positions on the basis of identical Normal distributions over end-of-period outcome, the markets' risk aversion is the market price of risk.

[22] The negative correlation between $M(a_{t+1}|a_t)$ and foreign lenders' payoff is ensured as long as $\sigma > 0$; in this case, a negative shock ε^a_{t+1} to future income decreases both the repayment probability and future prices, whereas it increases $M(a_{t+1}|a_t)$.

Domestic and foreign lenders' expected profits, respectively, are given by

$$
\begin{aligned}
E_t p^H_{t+1} = & E_t\{M(a_{t+1}|a_t)\{d_{t+1}q^D_{t+1} + (1-d_{t+1})[\lambda + (1-\lambda)(z+q_{t+1})]\}\} b^H_{t+1} b_{t+1} \\
& - q^H_t b^H_{t+1} b_{t+1} \qquad (15a) \\
E_t p^F_{t+1} = & E_t\{(1+r)^{-1}\{d_{t+1}q^D_{t+1} + (1-d_{t+1})[\lambda + (1-\lambda)(z+q_{t+1})]\}\}(1-b^H_{t+1}) b_{t+1} \\
& - q^F_t (1-b^H_{t+1}) b_{t+1}, \qquad (15b)
\end{aligned}
$$

where q_t, q^H_t and q^F_t denote the unique price prevailing in secondary markets, the bid price for domestic lenders and the bid price for foreign lenders, respectively. Equations (15) show the relevant expected profits for both types of investors before participating in the auction. In period $t+1$, in the event of repayment, lenders get fraction λ of a maturing bond and, on the remaining fraction $1-\lambda$, they receive the coupon payment z. In addition, the fraction that remains outstanding is traded in secondary markets and its value q_{t+1} depends on the persistent component of the TFP shock next period and on the sovereign's outstanding debt next period. Notice that, given the recursive formulation of the government's problem, q_{t+1} depends on the same policy functions used by the government at period t for q_t. Moreover, in the event of default, lenders' payoff is equal to the bond price under default q^D_t, defined below. Finally, next period payoff is discounted by the discount rates $M(a_{t+1}|a_t)$ and $(1+r)^{-1}$, which represent the relevant bond pricing kernels for domestic and foreign lenders, respectively.

As both types of lenders are perfectly competitive, both constraints in Equations (14) are satisfied with equality at the same time. Hence, both domestic and foreign lenders are subject to a zero-expected profit condition:

$$
\begin{aligned}
q^H_t &= E_t\{M(a_{t+1}|a_t)\{d_{t+1}q^D_{t+1} + (1-d_{t+1})[\lambda + (1-\lambda)(z+q_{t+1})]\}\} \qquad (16a) \\
q^F_t &= E_t\{(1+r)^{-1}\{d_{t+1}q^D_{t+1} + (1-d_{t+1})[\lambda + (1-\lambda)(z+q_{t+1})]\}\}, \qquad (16b)
\end{aligned}
$$

thereby ensuring that both the auction and the bargaining process always occur in equilibrium.

Finally, the solution to the Nash bargaining game yields

$$
b^H_{t+1} = 1 - \zeta, \qquad (17)
$$

which links the optimal share of sovereign debt captured by financial intermediaries to their bargaining power. This specification then allows the parameter for foreign lenders' bargaining power ζ to explicitly target the average domestic share of sovereign debt.[23]

[23] Notice that this formulation is not subject to the curse of dimensionality, since it introduces domestic debt without requiring the introduction of an additional state variable.

3.4.2 Second stage: Secondary markets

In the second stage, as type-specific bond prices satisfy the zero-expected profit conditions and domestic and foreign lenders bargain the optimal allocation of sovereign debt, secondary markets open and the exchange of bonds between the pool of lenders and the government occurs. Since the bonds issued by the government represent a homogenous good, the law of one price holds in equilibrium. Given the regime of perfect competition in international capital markets, the overall market price of bonds must satisfy a zero-expected profit condition for the entire pool of lenders. Hence, the bond price prevailing in secondary markets converges towards a weighted average of the prices for the two groups of lenders:

$$q_t = b^H_{t+1} q^H_t + (1 - b^H_{t+1}) q^F_t, \tag{18}$$

where the weight on each price is the respective share of total sovereign debt purchased by the individual group of lenders.[24] Ultimately, the overall price q_t represents the average cost of borrowing and becomes the relevant price for the government's borrowing decisions. Notice that this specification for q_t closely reflects the process of price formation in actual issuances of sovereign bonds on primary and secondary markets. On primary markets, auctions typically start with different investors posting their individual profit-maximizing bids; thereafter, the sovereign issues the total amount of debt on the basis of the overall price, a weighted average of the different bid prices, which becomes the relevant price on secondary markets. Secondary markets also form the bond price under default q^D_t, which is given by

$$\begin{aligned} q^D_t = & q_t E_t\{(1-\theta)(1+r) q^{DD}_{t+1}\} + \\ & q_t E_t\{\theta(1-\delta)[d_{t+1} q^{DR}_{t+1} + (1-d_{t+1})[\lambda + (1-\lambda)(z + q_{t+1})]]\}, \end{aligned}$$

where, given b_{t+1}, q^{DD}_{t+1} is equal to q^D_{t+1} for $(1+r)b_{t+1}$ and q^{DR}_{t+1} is equal to q^D_{t+1} for $(1-\delta)b_{t+1}$.

Two considerations suggest that the price schedule should be bound. First, Hatchondo, Martinez and Sosa Padilla (2014) argue that, in a model with long-duration bonds with a positive recovery rate, the government may have incentives to issue large amounts of debt, hence strongly expanding consumption, before defaulting. Following Hatchondo, Martinez and Padilla (forthcoming) and Hatchondo, Martinez and Roch (2015), in order to avoid this problem, the sovereign is assumed to face a lower bound $\underline{q}$ on the issuing prices of bonds q^H_t and q^F_t, while the price of government debt can be lower than $\underline{q}$ on secondary markets.

[24] This result follows from the observation that the aggregate zero-expected profit condition for the pool of lenders is given by

$$\begin{aligned} & b^H_{t+1} q^H_t + (1 - b^H_{t+1}) q^F_t = \\ & \quad E_t\{\{d_{t+1} q^D_{t+1} + (1-d_{t+1})[\lambda + (1-\lambda)(z + q_{t+1})]\}[b^H_{t+1} M(a_{t+1}|a_t) + (1 - b^H_{t+1})(1+r)^{-1}]\}. \end{aligned}$$

and that, by the law of one price, $q_t = q^H_t = q^F_t$.

The value for $\underline{q}$ is chosen so as to avoid consumption booms before defaults.[25] Second, the sovereign faces an upper bound $\bar{q}$ on the secondary market bond price q_t. The value for $\bar{q}$ matches the maximum bond price available in an economy inhabited by perfectly-competitive risk-neutral international lenders, that is the inverse of the (gross) risk-free rate $1+r$. If there is no possibility of default, the unit price would be a constant $\bar{q}$ such that $\bar{q} = [\lambda + (1-\lambda)(z+\bar{q})]/[1+r]$, which implies $\bar{q} = [\lambda + (1-\lambda)z]/[\lambda + r]$. Since $q_t \leq \bar{q}$, it follows that an internal rate of return r_t^* which makes the present discounted value of the promised sequence of future payments on a unit bond equal to the unit price is never below r. Finally, since the model is solved at annual frequency, the difference between the per-period yield r_t^*, such that $q_t = [\lambda + (1-\lambda)z]/[\lambda + r_t^*]$, and r is the annualized government bond yield spread r_t.

3.5 General equilibrium and debt limit

The following definition formally establishes the (Markov perfect) general equilibrium of this model.

Definition 1. *Given any state (a_t, b_t) at period t, the recursive general equilibrium is defined as the set of policy functions for (i) households' consumption and labor supply, (ii) firms' labor demand, (iii) financial intermediaries' share of domestic debt, (iv) the government's debt supply, default option and tax rates, (v) foreign lenders' debt demand and (vi) value functions $\{V(a_t, b_t)$, $V^R(a_t, b_t)$ and $V^D(a_t)\}$, such that:*

- *Taking as given the government and lenders' policies, c_t satisfies households' budget constraint (2), h_t satisfies households' labor supply function (3) and w_t satisfies firms' labor demand function (7);*
- *Taking as given the private sector and foreign lenders' policies, d_t and b_{t+1} solve problems (10)-(11), and τ_t^c, τ_t^h and g_t satisfy the government's targeting rules and budget constraint (8a), (8b) and (9), respectively;*
- *Taking as given the government and the non-financial private sector's policies, b_{t+1}^H satisfies the solution to the Nash bargaining game (17), q_t^H, q_t^F and q_t are consistent with investors' zero-expected profit conditions (16) and (18).*

The definition below identifies the default set as a function of the outstanding amount of total sovereign debt.

Definition 2. *For any given amount of outstanding debt, the default set $\mathcal{D}(b_t)$ is defined as the set of TFP shocks a_t for which available debt contracts in international capital markets make it optimal for the sovereign to default. Formally,*

$$\mathcal{D}(b_t) = \{a_t : V^R(a_t, b_t) < V^D(a_t)\}. \tag{19}$$

[25] The yield to maturity implied by $\underline{q}$ is higher than the maximum yield to maturity at which any European government issued debt since 2008 (Trebesch and Wright, 2013) and is never binding in simulations.

The following definition establishes the forward-looking, probabilistic notion of debt limit contingent on the current state of the economy.

Definition 3. *The probability-ϕ debt limit $b_{t+1}(\phi)$ is defined as the maximum level of borrowing today that makes it optimal for the sovereign to repay the total amount of its outstanding debt tomorrow with minimum probability $1-\phi$ for any given choice of borrowing tomorrow. Formally,*

$$b_{t+1}(\phi) = \sup\{b_{t+1} : \Pr[V^R(a_{t+1}, b_{t+1}) < V^D(a_{t+1})] \leq \phi\} \tag{20a}$$

$$= \sup\{b_{t+1} : E_t\mathcal{D}(b_{t+1}) \leq \phi\}. \tag{20b}$$

According to Definition 3, the probability-ϕ debt limit $b_{t+1}(\phi)$ is the maximum level of borrowing today associated with a probability of default tomorrow smaller than or equal to ϕ. This definition of debt limit is state contingent, due to the persistence of the TFP process as well as the debt and default policy functions, which link the current state of the economy to next period's TFP shock, optimal borrowing level and default option. Moreover, this definition of debt limit is forward looking and probabilistic, as $b_{t+1}(\phi)$ depends on the expected realization for the TFP shock and the choice of default tomorrow. Finally, notice that this definition explicitly links the government's value associated with both options $V^R(a_{t+1}, b_{t+1})$ and $V^D(a_{t+1})$ to the maximum amount of borrowing it is committed to repay. Hence, the notion of debt limit herein proposed is inherently related to the government's *willingness* to repay. As such, this notion differs from other definitions used in the literature on debt limits and sovereign creditworthiness, mostly hinging on the government's *ability* to repay.[26]

From the probability-ϕ debt limit, it is possible to derive the current fiscal space associated with a probability ϕ of future default. Notice that, disregarding stock-flow adjustments, the government's debt adjustment corresponds to the decrease (increase) in the face value of debt from the current to the next period, that is $\Delta b_t = b_t - b_{t+1}$. Let the probability-$\phi$ debt adjustment $\Delta b_t(\phi) = b_t - b_{t+1}(\phi)$ denote the minimum adjustment ensuring a default probability smaller than or equal to ϕ. Then, the relevant measure of fiscal space may be computed as

$$f_t(\phi) = \Delta b_t - \Delta b_t(\phi), \tag{21a}$$

$$= b_{t+1}(\phi) - b_{t+1}. \tag{21b}$$

The probability-ϕ fiscal space $f_t(\phi)$ is the maximum (absolute) amount available to the government for expanding its borrowing level while maintaining the probability of default tomorrow below or at ϕ. Hence, $f_t(\phi)$ gauges the sovereign's leeway in maximizing households' welfare while ensuring debt sustainability.

[26] See, for instance, the works by Ghosh et al. (2013), Bi (2012) and Polito and Wickens (2015).

Given the recursive formulation of the government's policy functions for default, borrowing as well as taxation and public consumption, it is then possible to calculate the probability of default at different time horizons. Let ϕ_t^n denote the probability of default in any period from $t+1$ to $t+n$ without defaulting in any previous period and conditional on information available at t. Formally,

$$\phi_t^n \equiv \sum_{s=1}^{n} E_t(d_{t+s}|d^{t+s}=0) \tag{22}$$

$$= \sum_{s=1}^{n} E_t(d_{t+s}) \prod_{r=0}^{s-1}[1-E_t(d_{t+r})], \tag{23}$$

where d^{t+s} refers to the history of default decisions from t to $t+s-1$. Hence, $b_{t+1}(\phi_t^n)$ and $f_t(\phi_t^n)$ are the relevant measures of debt limit and fiscal space for forward-looking policy makers seeking to evaluate the sustainability of debt at different time horizons.

3.6 Default incentives with domestic sovereign debt

In a model with domestic sovereign debt, the government internalizes the effects of its debt and default policies on households' welfare by solving problems (10)-(12). To see how the presence of domestic debt affects the government's incentives to default, consider the case of one-period bonds (i.e. $\lambda = 1$). The proposition below formalizes the main result.

Proposition 1. *For all $b_t^1 \leq b_t^2$, if default is optimal for b_t^1, then it will be optimal for b_t^2, namely $\mathcal{D}(b_t^1) \subseteq \mathcal{D}(b_t^2)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \leq \frac{u_g(c_t,h_t,g_t)}{u_c(c_t,h_t,g_t)}$. Conversely, if default is optimal for b_t^2, then it will be optimal for b_t^1, namely $\mathcal{D}(b_t^2) \subseteq \mathcal{D}(b_t^1)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \geq \frac{u_g(c_t,h_t,g_t)}{u_c(c_t,h_t,g_t)}$.*

Proof. See Appendix A. □

The first part of Proposition 1 states that, due to the monotonicity of the utility function, the sovereign faces higher incentives to default for higher levels of aggregate debt, but, due to the concavity of the utility function, this result holds only if the share of debt held by the domestic private sector is smaller than the marginal rate of substitution between public and private consumption. In other words, rising amounts of aggregate debt imply an increasing number of states for a_t in which the value of default is higher than the value of repayment only for sufficiently high levels of private (relative to public) consumption or, equivalently, for sufficiently low levels of marginal utility of private (relative to public) consumption. In contrast, the second part of Proposition 1 states the opposite result according to which the value of staying in the contract increases with aggregate sovereign debt as long as the share of debt held domestically is greater than the marginal rate of substitution between public and private consumption.

As observed by Arellano (2008), higher incentives to default are associated with higher levels of debt because the value of staying in the contract is decreasing in b_t, whereas the value of default is independent of b_t. If default is preferred in a given state a_t for some level of debt b_t, the value of the contract is less than the value of default. As b_t increases, the value of the contract monotonically decreases, becomes even lower than before and so default will continue to be preferred, since the value of default remains constant. Nevertheless, differently from Arellano (2008), this result does not hold for any level of private (relative to public) consumption. To see this, notice that, to the extent that government bonds are held domestically, public liabilities represent private assets. Hence, delinquent debt affects private and public consumption in different ways: default on a unit of debt raises public consumption by $u_g(c_t, h_t, g_t)$ whereas it lowers private consumption by $\frac{1-\zeta}{1+\tau_t^c} u_c(c_t, h_t, g_t)$ units. Hence, default incentives increase with debt as long as the positive effect of default on public consumption is larger than its negative effect on private consumption. Notice that, due to the concavity of the utility function, this condition holds when private consumption is sufficiently high relative to public consumption. In this case, the net effect of default on aggregate welfare is positive since the marginal benefit on public consumption more than compensates its marginal cost on private consumption.

Therefore, the relationship between default incentives and additional borrowing essentially depends on the distribution of domestic resources between the private and the public sectors. Given households' preferences as well as a path of borrowing decisions and TFP shocks, the distribution of the economy's output between private and public consumption is essentially determined by the government's tax rates on consumption and labor income. *Ceteris paribus*, as tax rates decrease, public consumption declines compared to private consumption; hence, the marginal benefit of default for the former increases compared to its marginal cost for the latter. Therefore, additional borrowing raises default incentives as long as tax rates are sufficiently low. In previous literature (see, for instance, Cuadra, Sanchez and Sapriza, 2010), taxation contributes to debt sustainability only by increasing public consumption and thus reducing the marginal cost of honoring the government's obligations. In this model, domestic debt amplifies the positive externality of taxation on debt sustainability: the government's incentives to stay in the contract also increase because taxes decrease private consumption and, thus, increase the overall marginal cost of default. Importantly, the model may produce the counterfactual result according to which the marginal cost of default may offset its marginal benefit, thus determining negative net welfare effects of default: in this case, the probability of default *decreases* with debt.[27] Hence, an economy with high tax rates features a higher (lower) public (private) consumption, a lower (higher) marginal benefit (cost) of default on public (private) consumption, so that the government faces lower incentives to default and, thus, higher levels of sustainable debt compared to an economy with low tax

[27] Although possible in theory, this result never occurs with the plausible calibrations studied in this paper.

rates. Finally, the relationship between default incentives and additional borrowing also depends on the domestic share of aggregate sovereign debt: a higher share $1-\zeta$ reduces default incentives as it increases the marginal private cost of default relative to its marginal public benefit. The exact quantitative implications of domestic sovereign debt on default incentives require knowledge of the specific functional forms and are then analyzed in the numerical assessment of the model.

4 Policy experiments

Besides the baseline model, this paper explores alternative specifications for debt and fiscal policies and their implications for aggregate welfare and sovereign debt sustainability.

4.1 Fiscal rules and credibility

In a first extension of the baseline model, the government is assumed to interact with supranational fiscal authorities requiring compliance with fiscal rules. Fiscal rules are typically defined as numerical targets on fiscal aggregates expected to be in place over a long period, aimed at correcting distorted incentives in policy making by binding national fiscal authorities to medium-term objectives. In this extension of the model, the presence of credible fiscal rules imposed by supranational fiscal authorities allows for the possibility of roll-over crises, namely self-fulfilling defaults, via a signaling mechanism. The government's compliance with fiscal rules informs market expectations about sovereign credibility: as a signal of an imminent sovereign default, a deviation from fiscal rules triggers a run by creditors on the government's debt and causes its anticipated default, thus realizing market expectations. This signaling mechanism shows how supranational fiscal authorities and international investors may interact through fiscal rules and market perceptions and contribute to an effective market-based fiscal discipline.

Fiscal rules are typically considered as institutional mechanisms aimed at supporting fiscal credibility and discipline (see, for instance, Giavazzi and Pagano, 1990). Difficulties in identifying the effects of fiscal rules are well documented (see, for instance, Poterba, 1996 and Afonso and Hauptmeier, 2009). However, several studies on the effects of fiscal rules in the EU find that rules are essential means to inform market expectations about sovereign credibility, interpreted as the *perceived* probability of a government servicing its debt (see, for instance, Heinemann, Osterloh and Kalb, 2014, and Nerlich and Reuter, 2015). The credibility channel of fiscal rules may play a major role especially in institutional and economic communities where supranational fiscal authorities may not credibly enforce compliance through alternative methods, such as sanctions or outright exclusion of borrowing opportunities deviating from the applicable fiscal rules. In this case, the possibility of fiscal slippage is particularly relevant, as governments can *de facto* expand their liabilities beyond the levels implied by fiscal rules.

This observation carries important implications for the model. First, in contrast to previous studies on sovereign debt and default with fiscal austerity plans (Hatchondo, Martinez and Roch, 2015, and Arellano and Bai, 2016), the government is not assumed to be endowed with a commitment technology that forces compliance with the fiscal rules imposed by supranational fiscal authorities.[28] Moreover, similarly to Chatterjee and Eyigungor (2012), this extension allows for the possibility of self-fulfilling debt crises in a model with long-term debt. Yet, while their focus is on exogenous "sunspot run equilibria", in this model roll-over crises are endogenously triggered by a deviation from fiscal rules via a signaling mechanism that informs market perceptions about the government's credibility. More specifically, if the government is subject to credit constraints – i.e. it would rather default than buy back debt from the market – and deviates from the applicable fiscal rules, foreign lenders receive a negative signal on the sovereign's credibility and fret over its willingness or ability to honor its obligations. As a result, creditors run on the government's liabilities and cause its default, thus realizing their own expectations. When "run equilibria" become more likely with higher deficits or debt levels, lower government bond prices reflect the increased probability of an imminent default. Therefore, as the sovereign seeks to stave off the risk of self-fulfilling debt crises and thus decrease its marginal cost of borrowing, fiscal rules foster market-based fiscal discipline via the credibility channel.[29]

On the basis of these observations, this model devises a signaling mechanisms linking a government's deviation from fiscal rules to increased market pressure via the credibility channel. Similarly to previous studies on sovereign debt and default with fiscal austerity plans, fiscal rules assume that the correction occurs through the government's debt policy, rather than its fiscal policy.[30] More specifically, the model investigates two rules, namely a deficit rule and a debt rule, closely reflecting the deficit criterion and the debt reduction rule, respectively, enshrined in the defining text of the EU fiscal governance framework, the Stability and Growth Pact (see European Commission, 2016).

First, the deficit rule requires the surplus s_t to be above a certain threshold:

$$s_t \geq \hat{s} y_t, \tag{24}$$

whereby the government commits to run a deficit-to-output ratio below the deficit ceiling

[28]When comparing model predictions with past experiences with fiscal rules, one should consider that previous studies are assuming certainty about the government's ability to commit to enforcing a rule, but such certainty has often been lacking in the past.

[29]Notice that, in a model with long-term debt, this mechanism rewards longer over shorter maturities through lower borrowing costs. In the context of sovereign borrowing, Cole and Kehoe (2000) argue that "run equilibria" are less likely if the sovereign issues long-term debt. With a large stock of long-term debt, the maturing portion of debt can be small, so that lenders' refusal to roll over does not significantly affect the borrower. Knowledge of this informs lenders' expectations and runs fail to be an equilibrium outcome.

[30]This assumption implies that the government may alter its issuance of debt more flexibly than its tax rates. However, the government cannot freely adjust its budget: if the required correction entails negative (public) consumption, then the government is assumed to default and suffer a TFP loss.

$-\hat{s} \in \mathbb{R}$ every period. The measure of surplus in the model needs to be consistent with surplus actually targeted by governments. To extract the model equivalent of the headline budget balance, or net government lending, considered by the EU fiscal governance framework, notice that, in a model with long-duration bonds and primary and secondary markets, the (negative) change in debt can be decomposed as follows from Equation (9):

$$b_t - b_{t+1} = \underbrace{\underbrace{\tau_t^c c_t + \tau_t^h w_t h_t - g_t}_{\text{primary surplus}} \quad - \quad \underbrace{(1-\lambda) z b_t}_{\text{interest payments}}}_{\text{headline surplus } (s_t)} \quad + \quad \underbrace{(1-q_t)[(1-\lambda)b_t - b_{t+1}]}_{\text{deficit-debt adjustments}}$$

where the first term corresponds to primary surplus (tax receipts minus public consumption), the second term refers to interest payments and the third term indicates deficit-debt (or stock-flow) adjustments due to market-to-face-value corrections.[31] Hence, the model equivalent of the headline budget balance is given by the difference between the first and the second term.

Second, conditional on the debt-to-output ratio exceeding the debt target $\hat{b}$, the debt rule requires that

$$b_{t+1} \leq b_t - \frac{b_t - \hat{b} y_t}{\hat{T}}, \tag{25}$$

whereby the government commits to reduce the part of its debt-to-output ratio in excess of $\hat{b} \in \mathbb{R}_+$ so as to reach the debt target within $\hat{T} - 1$ periods from t. Hence, $\hat{T} \in \{1, 2, ..., \infty\}$ gauges the degree of backloading required by supranational fiscal authorities, with $\hat{T} = 1$ implying a complete frontloading of the debt adjustment in the current period.[32] The two constraints (24) and (25) restrict the government's set of available debt contracts depending on the current state of the economy (a_t, b_t) and a set of "institutional" parameters $(\hat{s}, \hat{T}, \hat{b})$.

In what follows, the signaling mechanism is described. Consider a static coordination game played by the sovereign and foreign lenders at the start of any period in which the sovereign has positive outstanding debt and, conditional on honoring its obligations, desires to issue new bonds. The columns give the strategies of the sovereign and the rows give the strategies of lenders. If lenders purchase the new bond (B) and the sovereign repays its existing debt (R), the sovereign receives the payoff from repaying the loan and borrowing, denoted $V^+(a_t, b_t)$, and lenders earn a net return of 0 (i.e. lenders earn the risk-free return

[31] According to the European Central Bank (2014), general government debt (and therefore the change in debt) is recorded at face value, whereas financial transactions in the ESA 2010 are recorded at market value including accrued interest. In order to compensate for this difference in valuation, the deficit-debt adjustment includes the market-to-face-value adjustment as one of the items contributing to valuation effects and other change in debt excluded from EDP deficit. The adjustment is calculated as face values minus market values and applies only to transactions – that is, to new borrowings and repayment or buying-in of debt at prices which differ from nominal value (issuances and redemptions below or above par).

[32] Further, notice that the convergence path required by the debt rule does not entail the actual achievement of the debt target, but only a gradual (and endless) adjustment towards it.

$M(a_{t+1}|a_t)$ which is also the opportunity cost of their funds). If lenders purchase (B) and the sovereign defaults (D), the new bond is returned to the issuer and earns no interest, whereby lenders incur the (discounted) loss of interest earnings $r\hat{M}_{t+1}\Delta_t$, where $\hat{M}_{t+1}$ is the weighted average of domestic and foreign lenders' discount factors and Δ_t is the amount of new lending, and the sovereign receives $\bar{V}^D(a_t, b_t)$. If lenders do not purchase the new bond (N) even if the sovereign repays (R), the sovereign receives $V^-(a_t, b_t) \leq V^+(a_t, b_t)$ and lenders earn 0. Finally, if lenders do not lend (N) and the sovereign defaults (D), the payoffs are 0 and $\bar{V}^D(a_t, b_t)$ for lenders and the sovereign, respectively.

	R	D
B	$0, V^+(a_t, b_t)$	$-r\hat{M}_{t+1}\Delta_t, \bar{V}^D(a_t, b_t)$
N	$0, V^-(a_t, b_t)$	$0, \bar{V}^D(a_t, b_t)$

Further, assume that the sovereign always repays if it is indifferent between repaying and defaulting and lenders always purchase if they are indifferent between purchasing and not purchasing. Hence, this game has the following set of Nash equilibria, depending on the value of $\bar{V}^D(a_t)$. When $\bar{V}^D(a_t) \leq V^-(a_t, b_t) \leq V^+(a_t, b_t)$, the unique equilibrium is (B, R); if $V^-(a_t, b_t) \leq V^+(a_t, b_t) < \bar{V}^D(a_t)$, the unique equilibrium is (N, D); and if $V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t)$, both (B, R) and (N, D) are equilibria of the game. In this case, the realized equilibrium depends on the government's compliance with the fiscal rules in force. If the sovereign complies with the fiscal rule, the (B, R) equilibrium occurs; otherwise, the (N, D) equilibrium is selected. The latter case corresponds to a self-fulfilling debt crisis, whereby the government is credit constrained and deviates from fiscal rules, so that lenders receive a negative signal about the sovereign's credibility and run on its outstanding obligations.

This game requires the modification of the recursive equilibrium described above as follows. Let $U(a_t, b_t)$ denote the lifetime utility of the sovereign, depending on the latter's compliance with the applicable fiscal rules. Then,

$$V^+(a_t, b_t) = \max_{b_{t+1}} \{u(c_t, h_t, g_t) + \beta E_t[U(a_{t+1}, b_{t+1})]\}$$

subject to the same equilibrium conditions as problem (11). If there is no b_{t+1} such that the arguments of the current utility functions are nonnegative, then $V^+(a_t, b_t)$ is assumed to be equal to $-\infty$. Moreover,

$$V^-(a_t, b_t) = \max_{b_{t+1}} \{u(c_t, h_t, g_t) + \beta E_t[U(a_{t+1}, b_{t+1})]\}$$

subject to the same equilibrium conditions as problem (11) and $b_{t+1} \leq (1 - \lambda)b_t$, so that only nonpositive net issuance of new bonds is considered. Again, if there is no b_{t+1} such that the arguments of the current utility functions are nonnegative, then $V^-(a_t, b_t)$ is assumed to be equal to $-\infty$. Notice that $V^-(a_t, b_t) \leq V^+(a_t, b_t)$, unless the government finds it optimal to issue new bonds, in which case $V^-(a_t, b_t) < V^+(a_t, b_t)$. In addition, the value under default is similarly defined as

$$\bar{V}^D(a_t, b_t) = u(c_t, h_t, g_t) + \beta E_t[(\theta U(a_{t+1}, (1-\delta)(1+r)b_t) + (1-\theta)\bar{V}^D(a_{t+1}, (1+r)b_t)].$$

Finally, the continuation value for the sovereign is given by

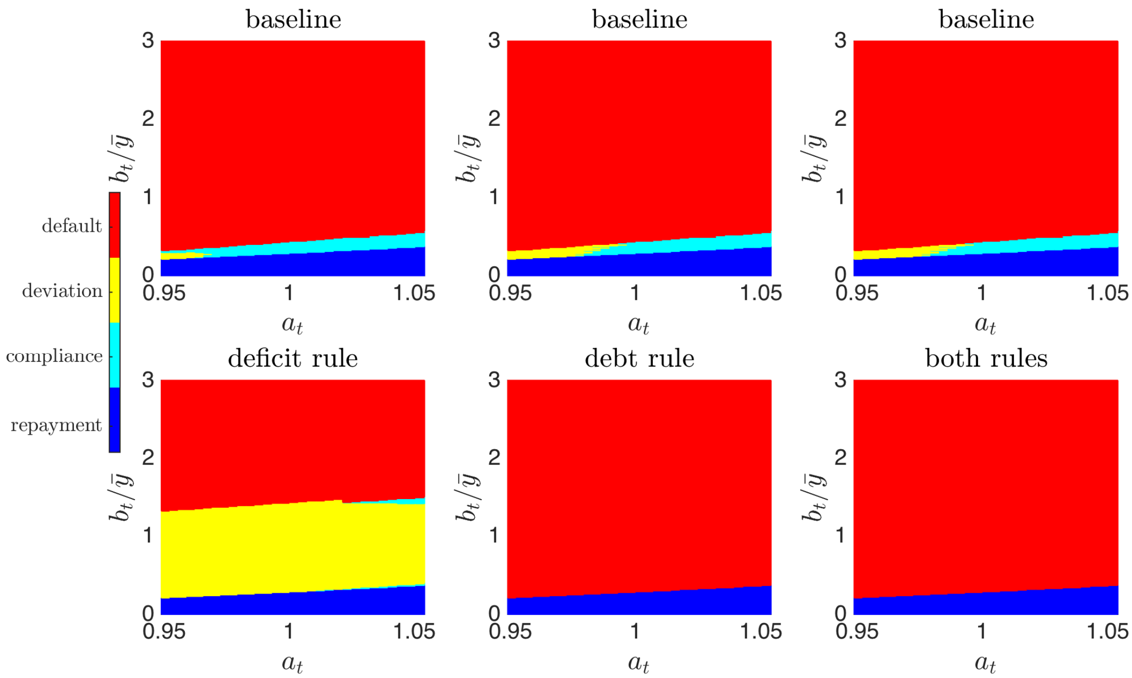
$$U(a_t, b_t) = \begin{cases} V^+(a_t, b_t) & \text{if } \bar{V}^D(a_t) \leq V^-(a_t, b_t) \quad (a) \\ \bar{V}^D(a_t) & \text{if } V^+(a_t, b_t) < \bar{V}^D(a_t) \quad (b) \\ V^+(a_t, b_t) & \text{if } V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t) \text{ and } k_t = 1 \quad (c) \\ \bar{V}^D(a_t) & \text{if } V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t) \text{ and } k_t = 0, \quad (d) \end{cases} \tag{26}$$

where $k_t = 1$ if the government complies with all the applicable fiscal rules (24) and/or (25) and $k_t = 0$ otherwise. As a consequence, the policy functions for the borrowing level b_{t+1} and the zero-expected profit condition for the bond price q_t need to be updated consistently with problem (26). Notice that the probability of a future deviation from the fiscal rule $E_t(1-k_{t+1})$ depends on the current borrowing decision as well as the future TFP shock.[33] Hence, *ex ante* compliance with or deviation from the fiscal rule need not coincide with its *ex post* outcome. In other words, the government does not retain full control over the outcome of its decisions: anticipating this, when deficits or debt levels (and, hence, the risk of roll-over crises) are high, the sovereign may prefer larger fiscal consolidations compared to a situation without self-fulfilling defaults in order to reduce the cost of servicing its debt.

The numerical solution of the model sheds further light into the quantitative implications of the credibility channel of fiscal rules. Figure 4 obtains by solving the model with the calibration for the average euro area country reported in Table 1, while assuming impatient agents ($\beta = 0.85$), a high coupon rate ($z = 0.1$), one-period bonds ($\lambda = 1$) and no domestic debt ($\zeta = 1$) in order to create noticeable effects on the agents' optimal policies. Figure 4 shows the occurrence of repayment (i.e. case (a) in Equation (26) above), compliance (i.e. case (c) above), deviation (i.e. case (d) above) and outright default (i.e. case (b) above) in the state space for TFP and debt. Each chart on the upper row refers to the baseline model, where only outright default affects the solution, and shows the states where roll-over crises would occur in case the government deviated from the fiscal rule(s) applicable in the model reflected in the adjacent chart on the lower row. The chart on the lower row refer, from left to right, to the extension of the baseline model where only the deficit rule, only the debt rule and both rules are enforced. Under this specification of model parameters, fiscal rules tighten

[33] In Chatterjee and Eyigungor (2012), the occurrence of self-fulfilling roll-over crises depends on the realization of an i.i.d. sunspot variable, so that "run equilibria" realize with a constant probability independent of income fluctuations.

Figure 4: Occurrence of repayment, compliance, deviation and default in the state space



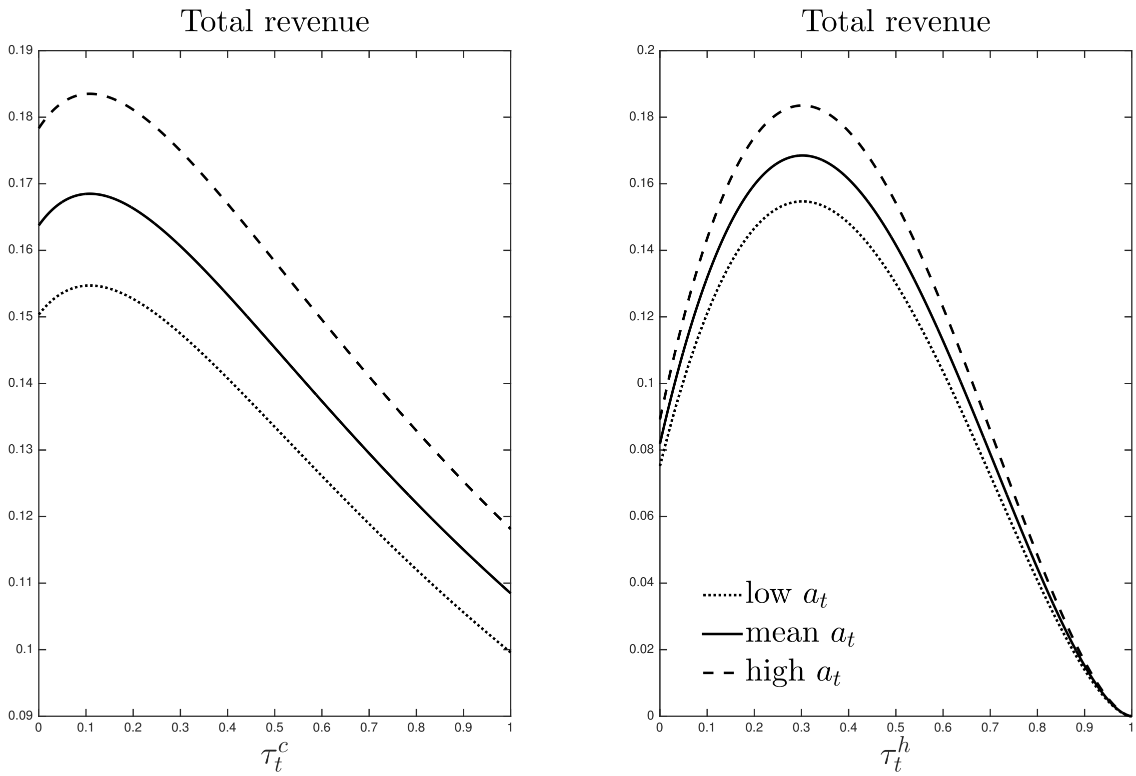
Notes: These charts are produced with the calibration reported in Table 1 except for four parameters, namely $\beta = 0.85$, $z = 0.1$, $\lambda = 1$ and $\zeta = 1$.

the borrowing constraint faced by the government as states with the possibility of a roll-over crisis in the baseline model transform into states with realized roll-over crises – in the model with only the deficit rule – or outright defaults – in the models with only the debt rule and both rules.

However, the benchmark calibration for the average euro area country does not exhibit such noticeable effects.[34] The credibility channel of fiscal rules may have little additional impact because the gap between $V^+(a_t, b_t)$ and $V^-(a_t, b_t)$ is positive only when the sovereign wishes to increase its debt b_{t+1} to more than $(1-\lambda)b_t$. In general, the gap is positive when borrowing costs are low and the government has an incentive to issue new debt, which happens when output is high and outstanding debt is low. Since during such times the default cost is high and $\bar{V}^D(a_t, b_t)$ is low, the conditions for a roll-over crisis (which requires that $V^-(a_t, b_t) < \bar{V}^D(a_t) \leq V^+(a_t, b_t)$) rarely occur and the randomness introduced by fiscal rules bears little consequence. In any case, as argued by Chatterjee and Eyigungor (2012), the introduction of stochastic roll-over crises is essential to explain the benefits of long-term debt relative to short-term debt. If the sovereign is carrying a large amount of one-period debt, the gap between $V^+(a_t, b_t)$ and $V^-(a_t, b_t)$ is large since the full payment of a large amount of debt is very costly. Thus, it is much more likely that the conditions for a default triggered by a deviation from fiscal rules will be satisfied.

[34]Results for different models with the benchmark calibration are reported in Section 5 below.

Figure 5: Laffer curves for consumption and labor tax rates



Notes: The right-hand chart shows the Laffer curve for the tax rate on consumption τ_t^c at the revenue-maximizing tax rate on labor income τ_t^h. The left-hand chart shows the Laffer curve for the tax rate on labor income τ_t^h at the revenue-maximizing tax rate on consumption τ_t^c. The variables are shown at the steady-state level for a_t and two standard deviations on each side of $\bar{a}$.

4.2 Laffer curves and institutional constraints

In a second extension of the baseline model, tax rates are assumed to reach the peak of state-contingent endogenous Laffer curves, namely the level at which they maximise total revenues. Then, the government solves the following problem:

$$\max_{\tau_t^c, \tau_t^h} \{\tau_t^c c_t + \tau_t^h w_t h_t\}, \tag{27}$$

subject to Equations (2), (3) and (7). As shown in Figure 5, Laffer curves can be generated with respect to both the consumption and the labor tax rates. This result stems from the interaction between private consumption and working hours via the private sector's equilibrium conditions.[35]

[35]However, given the non-linearities implied by the equilibrium conditions, no functional forms can be derived for the revenue-maximizing tax rates. Hence, the tax rates associated with the peak of the Laffer curve are determined through a numerical solution.

As taxes and debt are substitute means for financing public expenditures, high tax revenues decrease incentives to accumulate debt, thus relaxing the government's borrowing constraint and producing low default probabilities and high bond prices with positive effects on debt sustainability. However, the impact of high taxes on welfare is uncertain and depends on the benefits of low interest rates and the implications on employment, output and consumption. These implications are further explored in the quantitative assessment of the model.

5 Quantitative assessment of the model

In this section, the benchmark calibration and results from the numerical evaluation of the baseline and extended models with policy experiments are presented. The functional form for households' momentary utility function is assumed to be

$$u(c_t, h_t, g_t) = \kappa \frac{\left(c_t - \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)^{1-\sigma}}{1-\sigma} + (1-\kappa)\frac{g_t^{1-\sigma}}{1-\sigma},$$

where κ denotes the relative weight placed by households on private consumption, σ the coefficient of relative risk aversion (or the reciprocal of the intertemporal elasticity of substitution), ν is the Frisch elasticity of working hours. In order to solve the model, the state space is discretized. The support for TFP shocks spans 31 grid points around the long-run mean of the original process. The state space for the government bond price includes 300 equally-spaced grid points between 0 and $3\bar{y}$, so that implications can be drawn for debt-to-mean output ratios up to 300%.

5.1 Calibration

Table 1 reports the values of the model parameters calibrated at an annual frequency. The sample covers 19 euro area countries between 1995 and 2015. Data are drawn from the Eurosystem (Government Finance Statistics), Eurostat (National Accounts) and DataStream. Parameters for the representative euro area country are computed by taking a simple average over the estimated country-specific parameters. Values for 25 parameters need to be determined, whereas the other parameters are derived from the equilibrium conditions in the default-free deterministic steady state of the model. The inverse of the intertemporal elasticity of substitution σ and the steady state level of labor supply $\bar{h}$ are set to 2 and 0.33, respectively, which are standard values in the macroeconomic literature. The long-run mean of the TFP process is normalized to 1. Further, the parameters defining the fiscal rules $\hat{s}$, $\hat{b}$ and $\hat{T}$ take on values -3%, 60% and 20 to reflect the deficit and debt criteria envisaged in the EU Stability and Growth Pact. Defining TFP as the detrended ratio of real GDP to hours worked, the AR(1) process is fitted to Equation (6), determining an AR coefficient $\rho_a = 0.38$ and the

Table 1: Calibration of model parameters in average euro area country

parameter	value	target
σ	2	standard macroeconomic literature
$\bar{a}$	1	normalization
$\bar{h}$	0.33	standard macroeconomic literature
$\hat{s}$	-0.03	EU fiscal governance framework
$\hat{b}$	0.60	EU fiscal governance framework
$\hat{T}$	20	EU fiscal governance framework
ρ_a	0.38	TFP AR coefficient
σ_a	0.0163	TFP shock standard deviation
β	0.96	private sector's average credit rate
r	0.03	1-year Euribor
θ	0.33	expected duration of economic adjustment programme
ω_1	-0.80	1% TFP loss at lower bound
ω_2	0.88	maximum TFP loss at upper bound
$\bar{\tau}^c$	0.27	tax rate on total consumption
$\bar{\tau}^h$	0.33	tax rate on total labor income
$\bar{g}/\bar{c}$	0.36	public-to-private consumption ratio
λ	0.17	average maturity of government liabilities
z	0.06	government implicit tax rate
δ	0.65	haircut on net present value of sovereign liabilities
ζ	0.55	share of total debt held by domestic residents
$\bar{r}$	1.00	highest observed interest rate
ψ_b^c	0.13	elasticity of consumption tax rate to debt
ψ_a^c	0.19	elasticity of consumption tax rate to TFP
ψ_b^h	0.09	elasticity of labor tax rate to debt
ψ_a^h	-0.13	elasticity of labor tax rate to TFP

standard deviation for TFP shocks $\sigma_a = 0.0163$. Households' subjective discount factor β matches the non-financial private sector's average credit rate, defined as the cost of borrowing for new short-term loans on households and non-financial corporations. The rate of return r on riskless assets is set to 3% due to the long-run average value for the 1-year Euribor. The probability θ of receiving an offer to settle for recovered debt is set to 0.33, which corresponds to the expected duration of macroeconomic adjustment programmes implemented in Europe since the start of the sovereign debt crisis.[36] The output cost of default $\omega_1 = -0.80$ targets a 1% TFP loss at the lower end of the state space, as this loss is estimated by Borensztein and Panizza (2009) to be the *direct* real output cost of default; ω_2 is then set to 0.88 so as to ensure

[36] In the context of EU financial assistance to requesting countries, economic adjustment programmes typically last roughly three years, as in the case of Ireland (2010-2013), Portugal (2011-2013) and Cyprus (2013-2016); the third programme for Greece started in 2015 and was set to end in 2018. The duration of financial autarky and debt renegotiations after sovereign defaults, in particular on external debt, has received considerable attention in the literature. For instance, for Argentina's default in 2001 the settlement with the majority of the creditors was reached in 2005. In the default episodes of Russia (1998), Ecuador (1999) and Ukraine (1998), the renegotiation process lasted 2.3, 1.7 and 1.4 years, respectively, according to Benjamin and Wright (2009). In general, domestic debt restructuring periods tend to be not as long as in the case of external debt. For example, as documented by Sturzenegger and Zettelmeyer (2007), after the default by Russia in 1998 it took six months to restructure the domestic GKO bonds.

that $\omega(a_t)$ is increasing in a_t and reaches a maximum at the upper end of the state space. Similarly to Mendoza, Razin and Tesar (1994) and Reicher (2014), the steady-state tax rate on consumption $\bar{\tau}^c = 27\%$ is calculated as the long-run average of the ratio between indirect taxes and the sum of private consumption and government intermediate consumption net of indirect taxes, while the steady-state tax rate on labor income $\bar{\tau}^h = 33\%$ is computed as the long-run average of the the ratio between the sum of direct taxes on households and enterprises and social contributions net of subsidies and the sum of compensation of employees in the total economy and the gross operating surplus net of the numerator. The gross operating surplus is in turn defined as nominal GDP minus compensation of employees in the total economy minus indirect taxes plus subsidies. The steady-state public-to-private consumption ratio $\bar{g}/\bar{c}$ is set to 0.36 as the long-run average of the ratio of government consumption to private consumption. The probability λ of debt obligations maturing is 17%, targeting a long-run average maturity of total debt of 5.97 years. The coupon rate z is defined as the implicit tax rate on non-matured debt, that is the long-run average of the ratio of interest payments to $(1-\lambda)$ times outstanding debt at the end of the previous period. Following the empirical methodology in Sturzenegger and Zettelmeyer (2008), the actual default rate δ on the net present value of the government's total liabilities is set to 65%. Foreign lenders' bargaining power $\zeta = 0.55$ is determined as 1 minus the share of total debt held by domestic residents.[37] On the basis of arguments in the study by Trebesch and Wright (2013), the maximum interest rate charged on the sovereign takes on value 100%, higher than any interest rate actually ever observed in the data for euro area countries on relevant maturities. The elasticities ψ_j^l of tax rates for $l \in \{c, h\}$ and for $l \in \{c, h\}$ are fitted to Equations (8), where the constant critical level $\bar{b}$ for debt is set to the default-free steady-state level of debt.[38] Given the estimates of these 25 parameters, the remaining parameters obtain via the default-free deterministic steady-state of the model economy. Most notably, the values for the Frisch elasticity of labor supply ν and the weight of private (relative to public) consumption κ are 1.70 and 0.26, respectively, within the range of values commonly found in the macroeconomic literature. Furthermore, the maximum price $\bar{q} = 1.11$ implies a negative risk-free yield to maturity of -9.6% on 6-year government

[37] When time series for the share of total government debt held by domestic residents are not available (as in the case for Ireland, Luxembourg and Malta), they are proxied by the share of total securities other than shares issued by the government held by domestic monetary and financial institutions.

[38] Manipulating the government's budget constraint (9) yields the steady-state condition $\bar{b} = [\bar{\tau}^c\bar{c} + \bar{\tau}^h\bar{a}\bar{h} - \bar{g}]/[(1+\bar{q}-z)\lambda + z]$. Alternatively, the critical level $\bar{b}$ could be equal to $\hat{b}\bar{y}$ (60% of GDP) following extensive empirical research on targeting rules for fiscal policy in the European context. In a DSGE model for fiscal policy evaluation, Pappa and Vassilatos (2007) set the steady-state level of debt-to-GDP ratio to 60% to match the long-run average ratio for France and Germany between 2000 and 2005. In their empirical investigation of the determinants of government's fiscal behaviour for EU countries, Afonso and Hauptmeier (2009) evaluate the effects of fiscal rules against relevant benchmarks and find that, when the debt-to-GDP ratio is below the debt thresholds of 60%, 70% or 80%, a stronger overall fiscal rule contributes to improve the primary budget balance. In their estimates of fiscal reaction functions for the EU, Plödt and Reicher (2014), set the critical level for debt to reflect the 60% debt limit laid out by the Stability and Growth Pact and find that a fiscal rule that encourages a strong reduction in debt levels within twenty years would result in substantial pressure to run large primary surpluses for some countries.

bonds;[39] the minimum price $\underline{q} = 0.19$ implies a maximum yield to maturity of 434%, which rarely binds in numerical solutions but effectively limits the occurrence of consumption booms before default.

5.2 Model mechanics

The model is solved numerically via simultaneous value function and bond price function iterations.[40] The convergence issues due to the presence of multiple Markov perfect equilibria reported by Chatterjee and Eyigungor (2012) are negligible once a tie-break rule for the optimal debt policy is applied. According to this rule, the optimal debt policy b^i_{t+1} at iteration i cannot be higher than b^{i-1}_{t+1}. As iterations are initialized assuming a nil value function next period and a price function at its risk-free level, this rule may be interpreted as a backward solution in a finite-horizon environment from the last period, in which the highest amount of debt is chosen (as no default risk is charged on the sovereign borrower), to the current period, in which the sovereign internalizes the costs of default and opts for lower debt levels. This approach retains considerable computational advantages, as it does not entail the introduction of an additional state variable to allow for the "randomization" of the optimal debt policy.[41]

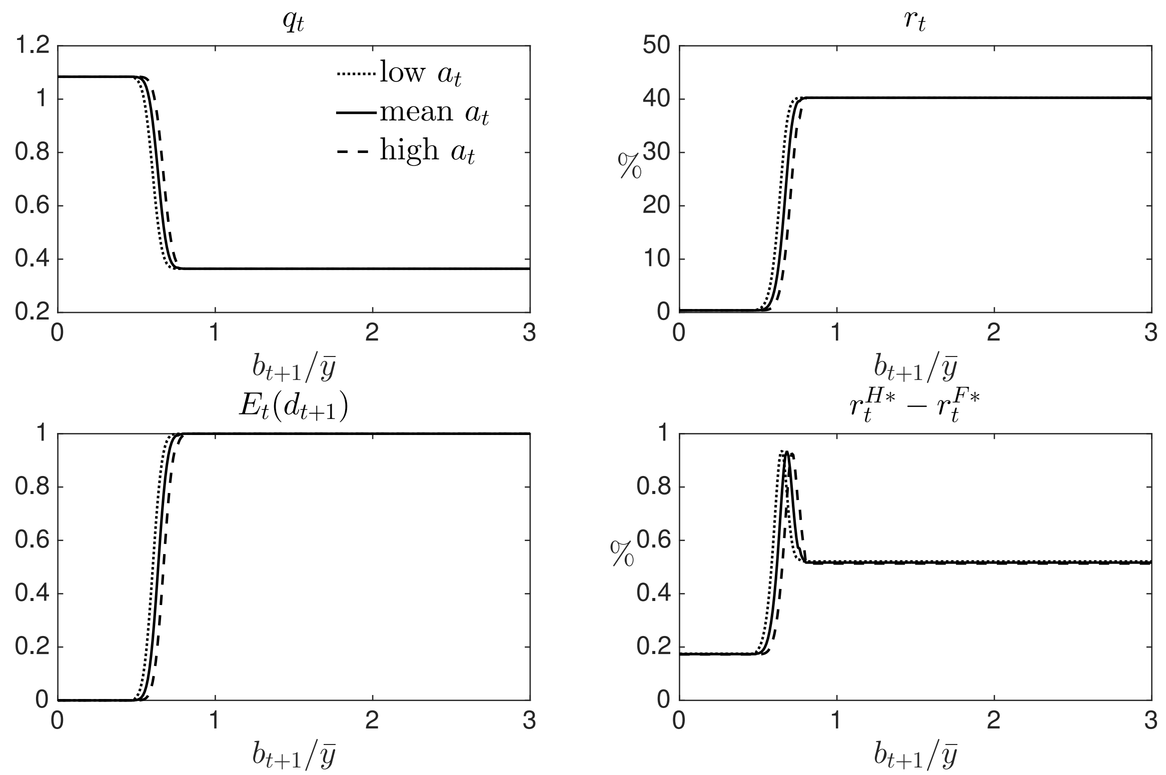
As shown in Figure 6, the bond price schedule q_t and the associated per-period spread r_t exhibit the common patterns observed in models with both strategic and non-strategic sovereign default (see, Arellano, 2008, and Bi, 2012). The price (spread) decreases (increases) as borrowing levels rise and productivity shrinks (top charts). This pattern is typically associated with the probability of default next period $E_t(d_{t+1})$, which increases as the government builds up leverage and during recessions (bottom left-hand chart). However, three additional factors drive interest rate dynamics. First, the presence of long-duration bonds drives a wedge between interest rates and default probabilities, as lenders understand that the sovereign's optimal decision next period is to take on a significant amount of debt even when the sovereign borrows a very small amount in the current period. Therefore, lenders' rational expectation to suffer a capital loss on the non-maturing portion of the debt further depresses prices even when the default probability is zero. Second, the existence of a recovery rate for debt implies that government bonds can be traded at a positive price q^D_t under default, as debt obligations grow at the risk-free rate while the country is in financial autarky, thus putting upward pressure on the secondary market price q_t. Third, spreads and default probabilities diverge

[39] This result stems from the relationship between the coupon rate z and the risk-free interest rate r, whereby high values for the former (due to high interest payments) and low values for the latter (due to monetary policy rates around the zero lower bound) entail negative interest rates paid on riskless long-duration bonds, consistently with empirical evidence (see, for instance, *Wall Street Journal*, June 14, 2016: German 10-Year Government Bond Yields Dip Below Zero as Brexit Fears Hit Market).

[40] This approach is shown by Hatchondo, Martinez and Sapriza (2010) to have considerable advantages over a solution algorithm with separate iterations for the value function and the bond price function in terms of computational time.

[41] This solution strategy bears similarities with several approaches in the literature on quantitative sovereign default (see, for instance, Arellano and Bai, 2014, and Hatchondo, Martinez and Roch, 2015).

Figure 6: Price and spread on secondary markets and spread between primary market bids

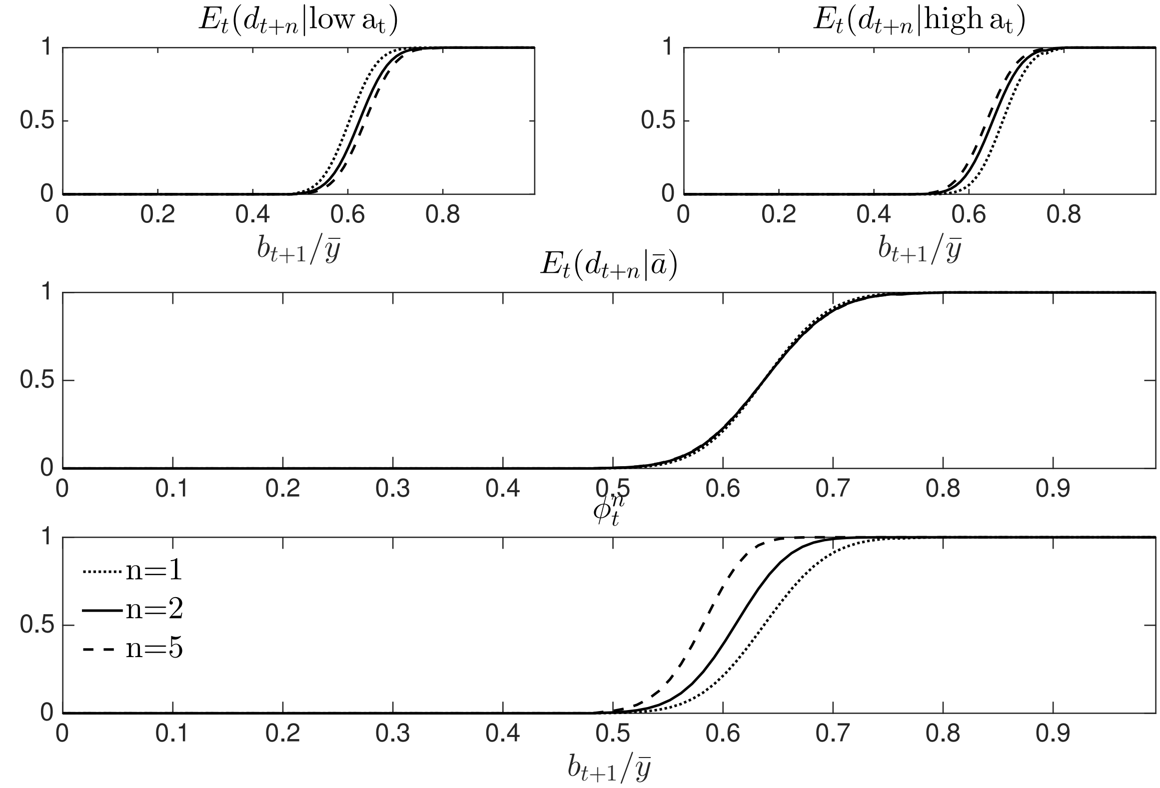


Notes: The variables are shown at the steady-state level for a_t and two standard deviations on each side of $\bar{a}$.

due to the presence of a pool of lenders composed of risk neutral foreign investors and risk averse domestic investors. Hence, risk aversion drives a wedge between the actuarially fair price charged by foreign lenders and the price requested by domestic investors. The spread between the interest rate charged by domestic and foreign lenders r_t^{H*} and r_t^{F*}, calculated as r_t^* but replacing q_t with q_t^H and q_t^F is depicted in the bottom right-hand chart. A persistent spread of 0.2% is charged as long as debt is perceived as safe, but it quickly rises up to almost 1% as borrowing becomes risky and then decreases to below 0.6% as the steepness of the price schedule becomes smaller. Notice that productivity significantly affects domestic investors' pricing paradigm only in the risky part of the sovereign's borrowing opportunities, whereby low TFP levels are associated with faster increases in interest rates as the sovereign borrows more compared to high TFP levels, since domestic investors factor in the negative effects of default on their expected income.

This model produces state-contingent estimates of debt limit distributions in an economy with stylized fiscal policy and strategic sovereign default decisions. Debt limit distributions essentially depend on two factors, namely the current state of the economy – via the equilibrium conditions of the model – and the default-free deterministic steady state of the economy – via the calibrated parameters. As regards the current state of the economy, the distributions

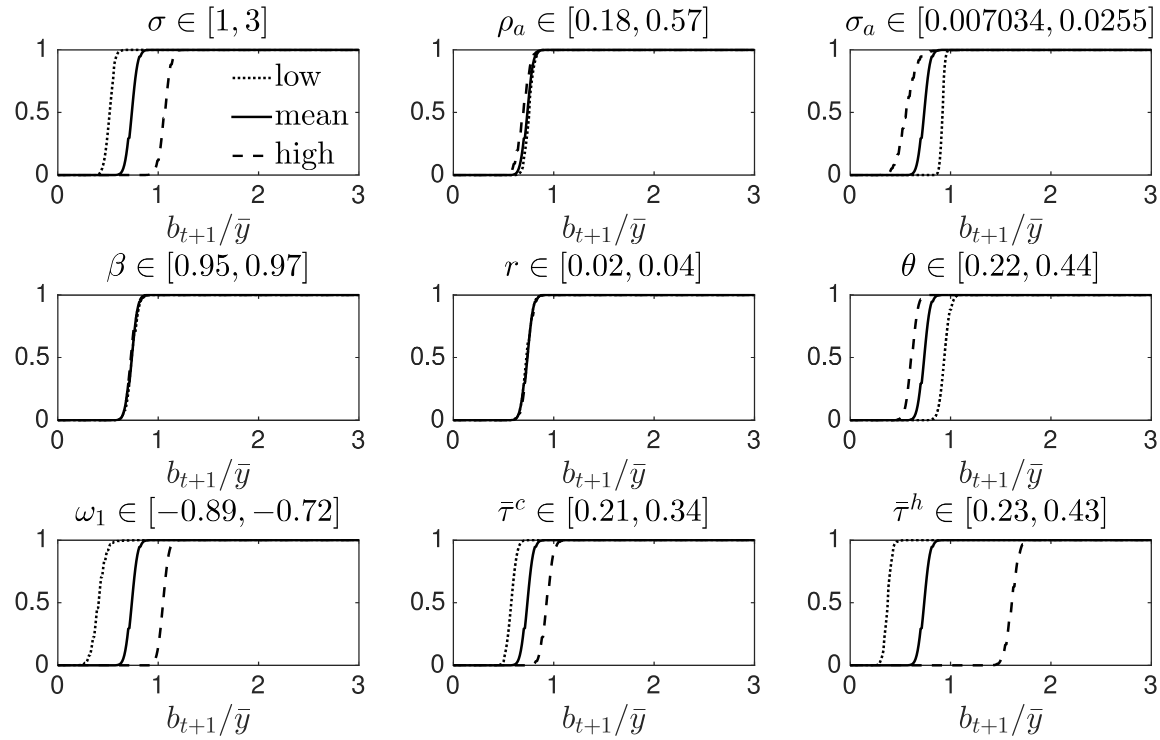
Figure 7: Default probabilities at different time horizons



of the debt limit $b_{t+1}(\phi_t^n)$ as a ratio of mean output can be observed from the probability of default ϕ_t^n) at different horizons and its determinants are shown in Figure 7. In the top charts, the probability $E_t(d_{t+n})$ of a default occurring exactly at time $t+n$ given information at t has different behaviors as the time horizon n increases depending on whether the economy is below or above its default-free steady-state TFP level $\bar{a}$ at t. Due to the mean reversion property of the stochastic process for TFP, negative (positive) shocks are expected when TFP is above (below) $\bar{a}$, so that TFP is expected to return to its long-run mean. Hence, $E_t(d_{t+n})$ decreases at longer horizons during recessions due to the expectation of an economic recovery (whereas the opposite is true when the economy is growing). In the case the economy is close to its long-run mean (second row in Figure 7), the distribution becomes slightly more disperse without noticeable effects on its expected value reflecting increased uncertainty as agents look farther into the future. As expected, the probability ϕ_t^n of default between $t+1$ and $t+n$ given no default in any period $s \in \{t, ..., t+n-1\}$ rises with n for any borrowing level, due to the increasing odds of a sovereign reneging its debt contracts as the time horizon expands.

As regards the deterministic steady state of the economy, a sensitivity analysis reveals the crucial role of some parameters in determining the results of the model. Figures 8 and 9 show the effect of different structural conditions on the debt limit distribution $b_{t+1}(\phi_t^n)$ for $n = 1$. The range of values under consideration spans two standard deviations around the

Figure 8: Sensitivity analysis of the probability of default



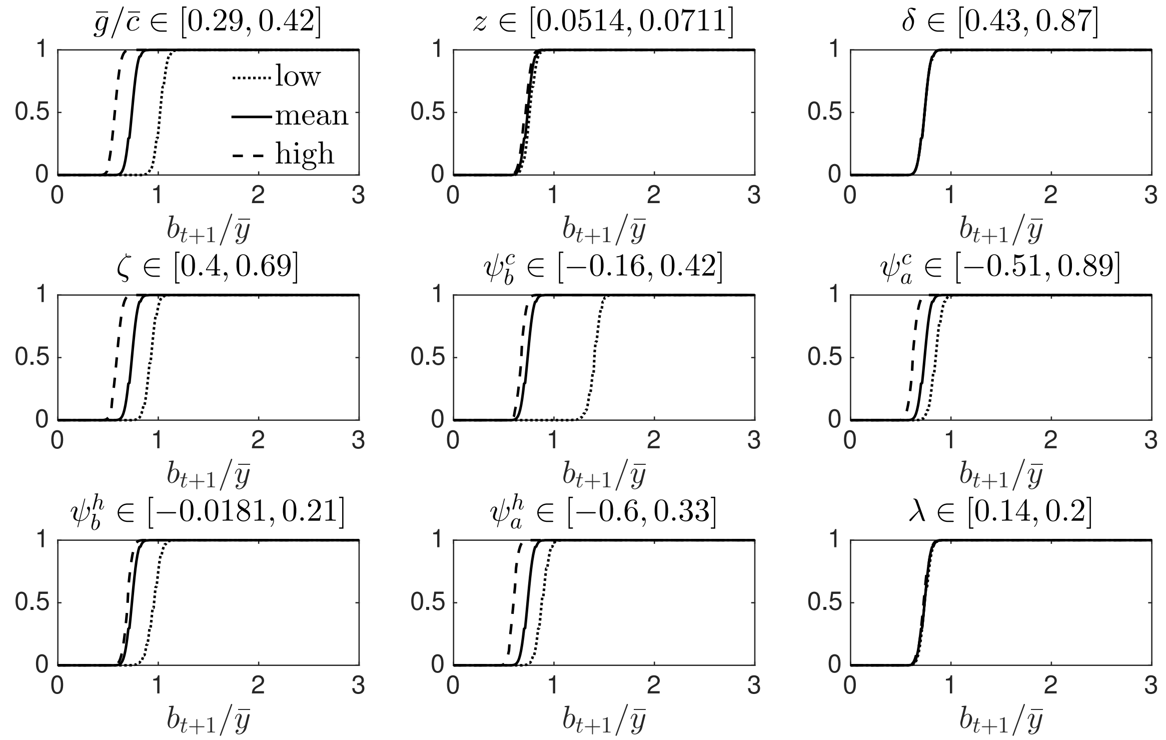
Notes: The default probabilities are shown at the steady-state level for a_t. The charts show debt limit distributions for the benchmark calibration (mean) and those implied by a calibration where only the analyzed parameter is changed over the grid of plausible values.

average of the cross section of parameter estimates for the countries in the sample, except for parameters common across countries, whose range is defined as the chosen value plus or minus one third. Hence, all the values studied in the sensitivity analysis lie within a set of plausible estimates. Most importantly, results should be interpreted in view of their significance for the selected sample of countries, while bearing in mind that setting parameters to values beyond the investigated range – some of which are often found in the related literature – may produce further changes to the debt limit distributions. The distributions for the benchmark calibration are compared against those implied by a different calibration of the model, where only one parameter is changed over the grid of plausible values.

As shown in Figures 8 and 9, some parameters do not have a sizeable impact on the distribution of $b_{t+1}(\phi_t^1)$, notably ρ_a, β, r, z, δ and λ. The most relevant parameters may be grouped as follows.

Riskiness and uncertainty. As domestic households' risk aversion σ increases, the distribution for the debt limit shifts rightward: since households become more risk averse, consumption smoothing through sovereign debt is valued more and, in turn, the gov-

Figure 9: Sensitivity analysis of the probability of default (continued)



Notes: The default probabilities are shown at the steady-state level for a_t. The charts show debt limit distributions for the benchmark calibration (mean) and those implied by a calibration where only the analyzed parameter is changed over the grid of plausible values.

ernment's borrowing opportunities increase. Furthermore, given a certain degree of risk aversion, a higher standard deviation of TFP shocks σ_a increases uncertainty; as the likelihood of tail events increases, higher TFP volatility makes default more frequent, especially at relatively low debt levels, and it translates into lower and more disperse debt limit distributions.

Cost of default. Parameters linked to the direct cost of default, such as the probability θ of receiving a settlement offer over recovered debt and the outright TFP loss of default ω_1, have similar effects: if θ increases or ω_1 decreases, then default becomes relatively less costly, as the economy spends shorter spells under financial autarky or it experiences a smaller contraction in productivity, respectively; hence, the sovereign faces lower incentives to honor its obligations and its debt limit distribution shifts leftward.[42]

Aggregate welfare A set of parameters refers to the weight of the public sector's utility

[42] To see why lower values for ω_1 are associated with lower costs of default, notice that $\omega_1 = (\iota - a^1/2a^N)/(1 - a^1/2a^N)$, so as to target a ι% TFP loss at the lower end of the state space for $a_t \in A = \{a^1, ..., a^N\}$. Clearly, ι is directly related to ω_1. Then, since $\omega_2 = (1 - \omega_1)/2a^N$, the loss function $\omega(a_t)$ is increasing in its argument and reaches a maximum at a^N, so that a higher cost at a^1 translates into a higher cost at all $a_t \in A$.

relative to the weight of the private sector's utility on aggregate welfare, namely $\bar{\tau}^c$, $\bar{\tau}^h$, $\bar{g}/\bar{c}$, ζ and ψ_j^l for $j \in \{a, b\}$ and for $l \in \{c, h\}$. The effects of different parameter values can be understood in light of Proposition 1 and its implications for the aggregate welfare of the economy by comparing the marginal private cost and public benefit implied by the sovereign's enforcement of its option to default.

Steady-state tax rates. Higher $\bar{\tau}^c$ and $\bar{\tau}^h$ increase the resources available to the government to repay its debt and, thus, expand its borrowing capacity; yet, this effect holds only as long as tax rates $\bar{\tau}^c$ and $\bar{\tau}^h$ are below the values corresponding to the peak of the Laffer curve; most importantly, higher tax rates imply a transfer of resources from the private to the public sector and, accordingly, an increase in the marginal utility of private consumption relative to public consumption; thus, the marginal cost of default on private consumption becomes higher, as opposed to the marginal benefit of default on public consumption; eventually, the sovereign's default incentives decrease and its borrowing opportunities rise.

Public-to-private consumption ratio. As $\bar{g}/\bar{c}$ increases, the steady-state marginal rate of substitution between public and private consumption decreases and so does the relative weight κ on the utility of private consumption, which is negatively affected by a domestic default; thus, as incentives to repudiate debt increase, the government faces tighter borrowing constraint.

Foreign lenders' bargaining power. A lower ζ determines a higher domestic share of total debt, so that a sovereign default imposes a relatively higher loss on the private sector's consumption, thus becoming relatively more costly for aggregate welfare; hence, the reduction in default incentives translates into higher borrowing opportunities.

Debt elasticities of tax rates. Lower (or even negative) ψ_b^l for $l \in \{c, h\}$, corresponding to a generally more unsustainable tax policy, deteriorate the credit conditions for the sovereign borrower, but only at sufficiently high debt levels; conversely, the government's borrowing capacity increases at low debt levels since a lower (or less positive) elasticity implies a lower decrease (or a larger increase) in the tax rate for debt b_t *below* the critical level $\bar{b}$ and a higher transfer of resources from the private to the public sector; as usual, a lower private (relative to public) consumption implies a higher marginal private cost (relative to the marginal public benefit) of default and higher incentives for the government to service its debt for sufficiently low debt levels; in the explored deviations from the benchmark calibration, this improvement outweighs the tightening of the borrowing constraint at relatively high debt levels, so that the overall effect is a rightward shift in the debt limit distribution.

TFP elasticities of tax rates. Higher ψ_a^l for $l \in \{c, h\}$, corresponding to a more counter-cyclical (or less pro-cyclical) tax policy, decrease the set of borrowing opportunities available to the sovereign; this tightening of the government's credit constraint occurs as a higher (or less negative) elasticity implies a larger decrease (or a lower increase) in the tax rate for TFP a_t *below* the long-run mean $\bar{a}$ and a lower transfer of resources from the private to the public sector during recessions; again, a higher private (relative to public) consumption implies a lower marginal private cost (relative to the marginal public benefit) of default and a higher default probability during economic contractions; in the explored deviations from the benchmark calibration, this deterioration offsets the loosening of the credit constraint during economic expansions, so that the overall effect is a leftward shift in the debt limit distribution.

Ultimately, although results warrant caution in the calibration methodology, this analysis suggests that the model is sufficiently flexible, so that it can explain a wide range of economic conditions. A proper calibration may yield significant results through the estimation of realistic debt limit distributions for a specific country with specific structural features.

5.3 Macroeconomic stabilization and debt sustainability

This structural approach allows for the accurate assessment of debt limit distributions and associated fiscal space, so as to evaluate the ability of fiscal policy in acting as a stabilizer of economic fluctuations and, thus, cushioning the economy against shocks. This section presents a yardstick approach useful for policy makers seeking to identify meaningful thresholds for the riskiness of different levels of debt and a measure for welfare associated with them.

First, in order to assess the riskiness implied by borrowing levels in different sectors of the debt limit distribution, significant thresholds for ϕ_t^n should be identified. To this purpose, three levels for the default probability bear a particular significance. First, according to the Eurosystem credit assessment framework (so-called ECAF), a regulatory text aimed at mitigating the credit risk of collateral used in monetary policy operations, all assets accepted by the Eurosystem as eligible collateral must meet the minimum requirement of an assessment of a certain credit quality on a harmonised rating scale, corresponding to a probability of default over a one-year horizon of up to 0.40%. Second, credit ratings are perceived by investors and policy makers as suitable proxies for the probability of default. Assuming that a credit rating reflects exclusively the ability of a government to repay debt, Polito and Wickens (2014) use interpolation techniques to map credit ratings into default probabilities at different time horizons. On the basis of credit ratings and default probabilities available in Moody's (2011), they find that the minimum default probability associated with speculative grade securities issues by sovereign borrowers is 41.5% at the one-year horizon – and increasing at longer maturities. Updating their results on the basis of Moody's (2015), the corresponding figure decreases to 34.5%, mainly reflecting normalizing market conditions from the heights of the crisis. Hence, a conservative assumption is to consider sovereign debt securities featuring a speculative grade whenever the probability of default crosses the 30% threshold. Third, the natural benchmark to assess debt sustainability is the case in which the government is expected to repudiate its obligations with certainty, that is a 100% probability of default. Accordingly, the analysis below refers to the probability-ϕ debt limit and fiscal space associated with a certain horizon n and risk threshold w given information at t as $b^w_{t+n}(\phi^n_t)$ and $f^w_{t+n}(\phi^n_t)$, respectively, for $w \in \{L, M, H\}$, where L, M and H denote a low, medium and high level of riskiness depending on whether ϕ^n_t is below or equal to 0.40%, 30% or 100%, respectively.

Second, it is essential to identify a measure of the welfare gains from adjusting away from the current debt level within the spectrum of borrowing opportunities. With this purpose in mind, welfare gains are gauged as the constant proportional change in units of consumption that would leave a consumer indifferent between preserving the current debt level in the following period, such that $b_{t+1} = b_t$, and borrowing a different amount.[43] Consider the model economy subject to the possibility of a roll-over crisis triggered by the credit-constrained sovereign deviating from the either fiscal rule, so that the violation of either constraint (24) or (25) may trigger run equilibria. Let $U(a_t, b_t, b_{t+1})$ denote the value function given the current state of the economy (a_t, b_t) and a borrowing decision b_{t+1} when both rules are enforced. The welfare gain $W(b_{t+1})$ of adjusting away from b_t towards b_{t+1} is computed as follows:

$$W(b_{t+1}) = \left(\frac{U(a_t, b_t, b_{t+1})}{U(a_t, b_t, b_t)}\right)^{\frac{1}{1-\sigma}} - 1. \tag{28}$$

By definition, the welfare gain $W(\tilde{b}_{t+1})$ under the optimal debt policy $\tilde{b}_{t+1}$ exceeds the welfare gain $W(b_{t+1})$ implied by any other borrowing decision. Moreover, notice that, as the specification of the utility function values consumption smoothing, the proposed measure of welfare is inherently linked to the benefits of macroeconomic stabilization on the overall economy.

Table 2 reports debt limits $b^w_{t+n}(\phi^n_t)$ and fiscal spaces $f^w_{t+n}(\phi^n_t)$ for $w \in \{L, M, H\}$ and $n \in \{1, 2, 5\}$ years in Panels A and B, respectively; Panel C shows the associated welfare gains $W(b^w_{t+n}(\phi^n_t))$ and sustainability gains, defined as the difference in the default probability implied by opting to remain at the current level of debt, so that $b_{t+1} = b_t$, and the default probability implied by moving to a different amount of borrowing, so that $b_{t+1} = b^w_{t+n}(\phi^n_t)$ for $n = 1$. The first column reports the optimal borrowing level $\tilde{b}_{t+1}$ (Panel A) and excess debt $b_t - \tilde{b}_{t+1}$ (Panel B) which separates the economy from reaching its welfare-maximizing level of debt. The figures in Table 2 refer to the benchmark calibration of the average economy.

As regards debt limits, Panel A shows how $b^w_{t+n}(\phi^n_t)$ increases with the level of riskiness w from $b^L_{t+1}(\phi^1_t) = 54\%$ to $b^H_{t+1}(\phi^1_t) = 86\%$ and decreases with the number of years n to $b^L_{t+5}(\phi^5_t) = 52\%$ (in terms of GDP). Although the presence of fiscal rules does not significantly affect the debt limit distribution, the optimal debt is lower whenever the model economy is

[43] This approach is common in the literature on quantitative sovereign debt and default. See, for instance, the works by Arellano and Bai (2016) and Hatchondo, Martinez and Roch (2015).

Table 2: Debt limit and fiscal space at different horizons and risk levels

Panel A. Debt limit – $b^w_{t+n}(\phi^n_t)$ (% of GDP)		1 year			2 years			5 years		
	Optimal	L	M	H	L	M	H	L	M	H
Baseline	76.21	53.75	64.34	85.53	52.47	62.20	85.53	51.76	59.77	85.50
Deficit rule	74.89	53.75	64.34	85.53	52.47	62.20	85.53	51.76	59.77	85.50
Debt rule	74.89	53.75	64.34	85.53	52.47	62.20	85.53	51.76	59.77	85.50
Both rules	74.89	53.75	64.34	85.53	52.47	62.20	85.53	51.76	59.77	85.50
Laffer curve	83.58	36.64	47.24	83.38	35.36	45.10	83.38	34.66	42.67	83.37
Panel B. Fiscal space – $f^w_{t+n}(\phi^n_t)$ (% of GDP)		1 year			2 years			5 years		
	Excess	L	M	H	L	M	H	L	M	H
Baseline	5.23	-27.69	-17.10	4.09	-28.97	-19.24	4.09	-29.68	-21.67	4.06
Deficit rule	6.55	-27.69	-17.10	4.09	-28.97	-19.24	4.09	-29.68	-21.67	4.06
Debt rule	6.55	-27.69	-17.10	4.09	-28.97	-19.24	4.09	-29.68	-21.67	4.06
Both rules	6.55	-27.69	-17.10	4.09	-28.97	-19.24	4.09	-29.68	-21.67	4.06
Laffer curve	-2.14	-44.80	-34.20	1.94	-46.08	-36.34	1.94	-46.78	-38.77	1.93
Panel C. Effects of adjustment from current debt level under both rules (%)		1 year			2 years			5 years		
	Optimal	L	M	H	L	M	H	L	M	H
Sustainability gain	5.00	99.60	70.00	0.00	99.60	70.00	0.00	99.60	70.00	0.00
Welfare gain	0.61	-1.54	-0.31	0.06	-1.82	-0.44	0.06	-2.00	-0.66	0.06

subject to any fiscal rule compared to the baseline economy. Thus, in the average euro area country, the possibility of rule-driven market crises deters the credit-constrained government from increasing its borrowing levels compared to a situation where no rules are enforced. When targeting rules for taxation policy are replaced with revenue-maximizing tax rates in the baseline economy, $b^w_{t+n}(\phi^n_t)$ decreases at every w and n. This counterintuitive result may be understood in light of Proposition 1 and its implications. Notice that the peak of the Laffer curve is reached at $\tau^c_t = 0.11$ and $\tau^h_t = 0.30$ (regardless of the TFP level), thereby implying tax rates lower than the steady-state levels in the baseline economy $\bar{\tau}^c = 0.27$ and $\bar{\tau}^h = 0.33$, respectively. Due to their effect on aggregate welfare, lower tax rates imply a lower marginal private cost of default relative to its marginal public benefit; as a result, the sovereign's default incentives increase and its borrowing opportunities shrink. In this light, as the optimal debt level $\tilde{b}_{t+1} = 84\%$ of GDP in the revenue-maximizing economy is above the unsustainable level of debt at any horizon, the government realizes that the highest welfare stems from stimulating public consumption through higher borrowing today and then causing a loss on the private sector by reneging outstanding debt obligations tomorrow.

As regards fiscal space, Panel B shows that $f^w_{t+n}(\phi^n_t)$ is positive only for $w = H$ at any n and under any model economy, so that the debt level in the average euro area economy $b_t = 81\%$ of GDP exhibits a high risk of default regardless of the presence of supranational fiscal authorities or a revenue-maximizing government. Consistently with the implications from the optimal debt policy, the government exhibits an excess debt under the baseline model and under any arrangement with fiscal rules, but it prefers increasing its leverage and defaulting on its obligations when tax receipts are at their maximum.

Finally, Panel C confirms the conventional wisdom suggesting that sustainability gains are typically associated with welfare losses from adjusting away from the current debt level. In the benchmark calibration of the average euro area country under both rules, as welfare gains increase from $W(b^L_{t+1}(\phi^1_t)) = -1.54\%$ to $W(b^H_{t+1}(\phi^1_t)) = 0.06\%$, sustainability gains decline from 99.60% to zero, whereby the economy featuring a probability of default close to 100% associated with an outstanding debt ratio of 81% seems to be bound to choose between certain losses or certain default. However, this pattern need not rule out the existence of a borrowing level ensuring positive gains in terms of both welfare and sustainability. In the model economy, welfare gains $W(b_{t+1})$ reach their peak with an increase by 0.61% in aggregate consumption when the government reduces its debt ratio from $b_t = 81\%$ to the optimal borrowing ratio $\tilde{b}_{t+1} = 75\%$. At the same time, this adjustment produces a sustainability gain of 5% through a reduction of the probability of default in the next period conditional on repayment in the current period. Therefore, when either fiscal rule may trigger a roll-over crisis for a credit-constrained sovereign borrower, the model economy may reap the benefits of debt reduction without forgoing welfare. Ultimately, as observed through the lenses of a general equilibrium model with strategic sovereign default, the average euro area country does not face a trade-off between macroeconomic stabilization and sustainability, as a debt-driven fiscal consolidation can reduce the probability of default while increasing welfare at the same time.

6 Conclusions

This paper analyzes fiscal space through the lenses of a dynamic general equilibrium model with strategic sovereign default. This paper contributes to the current debate on fiscal space in the euro area by analyzing and quantifying the implications of macroeconomic fundamentals and fiscal policy on a government's incentives to default and the implied debt limit distributions. Crucially, besides the quantification of debt limits, this approach enables welfare considerations and seeks to complement alternative non-structural approaches to analyzing fiscal sustainability, such as rules-based and DSA-based fiscal frameworks. The baseline model includes several features, such as fiscal policy instruments, risk averse investors, long-duration bonds with recovered debt as well as, via novel applications, domestic debt and fiscal rules. The analytical and numerical solution of the model shows that aggregate welfare, weighing the marginal private cost against the marginal public benefit of default, a major driver of a sovereign's incentives to honor its obligations. A sensitivity analysis within a range of plausible calibrations confirms this result and highlights the effects of parameters linked to uncertainty and default costs. In the benchmark calibration, the presence of fiscal rules does not worsen sustainability concerns as it does not affect the government's fiscal space; yet, the possibility of roll-over crises fosters market-based fiscal discipline by reducing the sovereign's optimal level of debt. The empirical application of the model finds that the average euro area country is currently facing high risk of default. However, the economy is not subject to the conventional trade-off between macroeconomic stabilization and sustainability. By implementing a debt-based fiscal consolidation, the average euro area country can simultaneously increase its welfare and reduce its sustainability concerns.

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Appendix A Proof of Theorem

In the following proofs, consider the case of one-period bonds (i.e. $\lambda = 1$) and let x_t^D denote the value of variable x_t under financial autarky.

Proposition 1. *For all $b_t^1 \leq b_t^2$, if default is optimal for b_t^1, then it will be optimal for b_t^2, namely $\mathcal{D}(b_t^1) \subseteq \mathcal{D}(b_t^2)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \leq \frac{u_g(c_t,h_t,g_t)}{u_c(c_t,h_t,g_t)}$. Conversely, if default is optimal for b_t^2, then it will be optimal for b_t^1, namely $\mathcal{D}(b_t^2) \subseteq \mathcal{D}(b_t^1)$, if and only if $\frac{1-\zeta}{1+\tau_t^c} \geq \frac{u_g(c_t,h_t,g_t)}{u_c(c_t,h_t,g_t)}$.*

Proof. The first part of Proposition 1 is proven as follows. For all $a_t \in \mathcal{D}(b_t^1)$, $V^D(a_t) = u(c_t^D, h_t^D, g_t^D) + \beta E_t[(\theta V(a_{t+1}, 0) + (1-\theta)V^D(a_{t+1})] > u(c_t, h_t, g_t|b_t^1) + \beta E_t[V(a_{t+1}, b_{t+1})] = V^R(a_t, b_t^1)$, given any b_{t+1}. Notice that

$$\frac{(1-\tau_t^h)w_t h_t + p_t^F + (1-\zeta)(b_t^1 - q_t b_{t+1})}{(1+\tau_t^c)} \leq \frac{(1-\tau_t^h)w_t h_t + p_t^F + (1-\zeta)(b_t^2 - q_t b_{t+1})}{(1+\tau_t^c)},$$

whereas

$$\tau_t^c c_t + \tau_t^h w_t h_t + q_t b_{t+1} - b_t^1 \geq \tau_t^c c_t + \tau_t^h w_t h_t + q_t b_{t+1} - b_t^2.$$

Hence, if and only if $\frac{1-\zeta}{1+\tau_t^c} \leq \frac{u_g(c_t,h_t,g_t)}{u_c(c_t,h_t,g_t)}$, then

$$\frac{\partial u(c_t, h_t, g_t)}{\partial b_t} = \frac{1-\zeta}{1+\tau_t^c} u_c(c_t, h_t, g_t) - u_g(c_t, h_t, g_t) \leq 0$$

so that the flow utility function is decreasing in b_t and

$$\begin{aligned} V^R(a_t, b_t^1) &= u(c_t, h_t, g_t|b_t^1) + \beta E_t[V(a_{t+1}, b_{t+1})] \\ &\geq u(c_t, h_t, g_t|b_t^2) + \beta E_t[V(a_{t+1}, b_{t+1})] \\ &= V^R(a_t, b_t^2). \end{aligned}$$

Thus, the value of the contract under repayment is decreasing in aggregate sovereign debt. Hence, $V^D(a_t) = u(c_t^D, h_t^D, g_t^D) + \beta E_t[(\theta V(a_{t+1}, 0) + (1-\theta)V^D(a_{t+1})] > u(c_t, h_t, g_t|b_t^2) + \beta E_t[V(a_{t+1}, b_{t+1})] = V^R(a_t, b_t^2)$, which implies that $a_t \in \mathcal{D}(b_t^2)$.

The second part of Proposition 1 is proven as follows. For all $a_t \in \mathcal{D}(b_t^2)$, $V^D(a_t) > V^R(a_t, b_t^2)$, given any b_{t+1}. If and only if $\frac{1-\zeta}{1+\tau_t^c} \geq \frac{u_g(c_t,h_t,g_t)}{u_c(c_t,h_t,g_t)}$, then $\frac{\partial u(c_t,h_t,g_t)}{\partial b_t} \geq 0$ so that the flow utility function is increasing in b_t and $V^R(a_t, b_t^2) \geq V^R(a_t, b_t^1)$. Thus, the value of the contract under repayment is increasing in aggregate sovereign debt. Hence, $V^D(a_t) > V^R(a_t, b_t^1)$, which implies that $a_t \in \mathcal{D}(b_t^1)$. □