Austerity to save the banks? A quantitative model of sovereign default with endogenous default costs and a financial sector

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Abstract

Why do states repay their debt? This paper develops a quantitative model of sovereign default with endogenous default costs to propose a novel answer to this question. In the model, the government ex ante has an incentive to borrow internationally due to a difference between the world interest rate and the domestic return on capital, which arises from a friction in the domestic banking sector. Since this sector is exposed to sovereign debt, sovereign default causes losses for the banks, which translate into a financial crisis. When deciding upon repayment ex post, the government trades these costs off against the advantage of not repaying international investors. Besides replicating cyclical moments, the model is able to generate not only output costs of a realistic magnitude, but also endogenously predicts that default is followed by a period during which no new foreign lending takes place. The duration of this period is in line with empirical estimates.

1 Introduction

In the recent European debt crisis many governments faced a dilemma: Enforce austerity in times of crisis to generate the surplus needed to repay their debt or default on their debt obligations and face the financial disruptions entailed. In many cases it seemed hard to believe that austerity policies were the less painful choice and some countries eventually defaulted, most notably Greece. Yet the Greek experience, just like the recent defaults by Russia or Argentina, confirmed that default does indeed entail a significant cost: In all three cases the financial sector suffered major losses, which lead to severe repercussions in the real economy.

These dramatic events brought an old question to the forefront of the policy and academic debate: Why should sovereign governments repay their debt, even though there is no institution that can force them to? This paper proposes a quantitative model of opportunistic sovereign default with endogenous default costs reflecting the recent experiences in Greece and elsewhere to answer this question.

In the model debt is issued by the government of a small open economy. This economy is populated by households, banks and firms, which produce output from labor and capital. The banks issue deposits to the household and invest into loans to firms and sovereign bonds. Their capacity to intermediate private savings within the economy is constrained by a financial friction, which limits their leverage. Besides the domestic banks, foreign investors are also active in the sovereign debt market. This debt is commitment free, that is the government, which maximizes the domestic households' utility, can default on its debt in a nondiscriminatory fashion whenever it finds it opportune to do so. Since this model does not rely on any exogenous costs of default, not repaying the foreign held debt per-se is a free lunch. Yet since defaulting on domestic banks generates a financial crisis in the domestic economy, default becomes costly to the benevolent government. Besides the output losses due to the financial crisis, default furthermore is followed by a period during which the government prefers not to issue new debt given the current prices. In other words, the government self excludes itself from international capital markets. The model is hence capable of simultaneously explaining both the output costs of default as well as the breakdown of international lending after default. These costs constitute a commitment device, which explains why governments mostly repay and hence why they *can* borrow. Furthermore, the existence of capital in combination with the frictions in domestic intermediation drive a wedge between the actual return on capital and the return perceived by the household. This gives rise to an arbitrage motive, which explains why governments *want* to borrow.

The model is calibrated to Greek data. Besides inheriting the quantitative features of the RBC production structure, the model is successful at replicating both the empirical magnitude of the output losses of default as well as estimates of the distribution of the duration of the international capital market shutdown. Furthermore, I use the model to argue that a recently debated policy proposal to reduce the exposure of European banks to domestic sovereign debt may have severe unintended consequences: As the banks exposure is reduced banks do not only become more resilient to sovereign debt crisis. Such a policy furthermore reduces the degree to which the government can commit repayment of its debt. Given a 20% reduction in the banks' exposure, the model predicts a reduction in the total amount of foreign debt sustainable by about one half. If implemented abruptly, this reform would trigger default with a high probability.

This theory is motivated by a large body of empirical evidence. De Paoli et al. (2006), Sturzenegger and Zettelmeyer (2007), Acharya et al. (2011), Bolton and Jeanne (2011), Reinhart and Rogoff (2011b), Sosa-Padilla (2012) and Gennaioli et al. (2014) all document the link between sovereign default and financial crisis. Gennaioli et al. (2014) for example compile an international panel comprising data on the macro economy, public debt and financial institutions' balance sheets, covering 110 default episodes. They document that domestic banks hold significant amounts of domestic sovereign debt and that defaults often precede financial crises. Furthermore, they provide evidence that credit extended by banks to the private sector drops significantly in case of default and that this drop is stronger, the more exposed banks are to sovereign debt. Consequentially, they find that higher shares of domestic debt reduce the probability of default.

This paper is related to a recent field of theoretical literature that analyzes sovereign default in quantitative models, based on the Eaton and Gersovitz (1981) framework. Motivated by empirical observations this literature initially simply *assumed* that default on external sovereign debt is costly, due to capital market exclusion and output costs (see Arellano (2008) or Aguiar and Gopinath (2006)). Subsequently, researchers attempted to endogenously *explain* the disincentives to default. Focusing on domestic debt, D'Erasmo and Mendoza (2012) show that the distributional effects of default can explain default and repayment, while Guembel and Sussman (2009) propose a theory of political support in favor of or against default. In Mendoza and Yue (2012) default on foreign debt leads to market exclusion not only for the government but also for importing firms, which generates output losses. This paper adds to this literature by proposing a quantitative model where the costs of default arise from the exposure of the financial system to government debt.

In doing so it provides 3 novel conceptual insights, all of which are found to be quantitatively plausible: First, the introduction of physical capital not only bridges the gap between the sovereign default literature and the RBC model[1], it furthermore yields a new theory for why governments borrow. Virtually all the quantitative external sovereign default literature to date relies on the assumption that the borrowing country is impatient relative to the world interest rate to explain why the country borrows. I show that capital as a domestic savings technology together with the financial friction implies a permanent difference between the return on domestic capital and the world interest rate, creating an opportunity for arbitrage. Second, this is the first model on sovereign default to generate market exclusion as an equilibrium outcome. The existing literature typically simply introduces exclusion by assumption, often referring to it as a coordinated punishment strategy by lenders[2]. In many cases where the output costs

[1] Joo (2014) and Gornemann (2015) also introduce capital into an Eaton and Gersovitz (1981) default model.

[2] Kletzer (1994) has criticized this argument, since it would be profitable for each individual lender to deviate from the punishment strategy.

of default are modeled endogenously, they actually result from this assumption (e.g. Mendoza and Yue (2012)). On the contrary, in the model I propose market exclusion arises as an optimal choice: During the financial crisis the marginal return on domestic investment is depressed and falls below the world interest rate. Hence, the gains from trade vanish temporarily, borrowing becomes relatively expensive and abstention from external borrowing is optimal. Third, this paper provides a new quantitative theory of the costs of sovereign default, where sovereign default leads to a breakdown of intermediation resulting in a period of underinvestment.

Five recent working papers also endogenize output costs by introducing a financial sector, many of which have been concurrent work. Like in this paper, in these models the bank is exposed to sovereign debt. Sosa-Padilla (2012) develops a very stylized model in which banks' wealth directly serves as an input to production, without providing intermediation services. In Engler and Grosse Steffen (2016), Mallucci (2014) and Perez (2015) local banks trade on inter-banking markets, and sovereign default brings these markets to collapse: in the first and second case because sovereign bonds are necessary as collateral to facilitate interbank lending and this collateral disappears in case of default, in the last because the interbank lending is leverage constrained. The collapse of the interbank markets then leads to efficiency losses. Finally, in Boz et al. (2014) leverage constraint banks intermediate households funds to firms. While similar in spirit, a number of features in my model are unique. First, all these models have in common that the bank needs to finance the labor bill[3] in advance through intra-period loans. In my model on the contrary, the bank finances intertemporal capital. This gives rise to a more realistic bank balance sheet and, as explained above, generates a novel motive for borrowing. Second, the model I propose gives rise to exclusion endogenously as a consequence of the financial crisis. On the contrary, all of these papers simply assume exclusion and, apart from Sosa-Padilla (2012), rely on this assumption to generate the financial crisis and the corresponding output costs of default.

The idea that sovereign default inflicts costly damage to the financial sector is also developed in a more stylized manner in 3 periods models like Acharya et al. (2011), Basu (2010), Bolton and Jeanne (2011), Brutti (2011), Gennaioli et al. (2014), Erce (2012) and Mayer (2011). Due to their 1 shot nature, none of these models can speak about market exclusion or be tested quantitatively.

At the same time this paper is also related to the macro literature on the role of frictions in the financial sector, for which the handbook chapter of Gertler and Kiyotaki (2010) is representative. Yet, while the shocks to the banks' balance sheets in these model result from the banks' lending to the real economy, I focus on the shocks resulting from their lending to the government. Boccola (2016) analyzes sovereign default and its consequences in the context of the former model, yet he considers the default decision as exogenous.

The rest of the paper is structured as follows: The next section lays out a quantitative model of sovereign default with a 3 sector production economy and discusses the key assumptions. Section 3 explains the mechanisms of the model, before the computational strategy and the

[3] In the last paper, firms furthermore need to finance some startup costs.

calibration are discussed in section 4. Section 5 presents the main quantitative results and section 6 uses the model to evaluate the effects of a policy that reduces the exposure of banks to domestic sovereign debt. The last section concludes.

2 The model

This chapter proposes a dynamic small open economy model of sovereign default, where the only costs of default stem from the exposure of the domestic banking sector to sovereign risk. The domestic private economy is made up of 3 sectors, namely households, banks and firms and is governed by a benevolent government. The domestic economy is open and can borrow from international lenders (the rest of the world).

2.1 The household

There is a continuum of mass 1 of identical households, which each make a consumption-labor-savings choice. The household is risk averse and can invest into bank deposits d_h and loans to firms k_h. Bank deposits are promises to pay 1 unit of the final good tomorrow, which can be bought at price r. Loans are promises to repay R' units of the final good tomorrow, which can be bought at price $1+\xi$, where ξ is an exogenous variable, which reflects transaction costs and which follows a stochastic process discussed later.[4] Both assets are safe and loans can't be shorted.[5] The household's disposable income is the sum of the value of his assets carried over from the last period $d_h + Rk_h$, his labor income Wl_h and the lump sum dividend and profit payments from the banks Div and the firms π, minus a lump sum tax T. The household has rational expectations and chooses labor, consumption and investment to maximize his lifetime utility, taking prices and aggregate states as given.

The household's problem hence is:

$$V_h(d,\Omega) = max_{c,d'_h,k'_h,l_h} u(c,l) + \beta E_{|\Omega}\, V_h(d',\Omega')$$

$$st.$$

$$c + rd'_h + (1+\xi)k'_h + T = d_h + Rk_h + Wl_h + Div + \pi \tag{1}$$

$$k'_h \geq 0$$

where Ω denotes the aggregate state vector. For instantaneous utility I choose a conventional CARA function with additively separable disutility from labor, as for example in Boccola (2016):

$$u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} + \chi\frac{l^{1+\nu}}{1+\nu}$$

[4]As will become clear later, the fact that banks do not face these transaction costs makes intermediated lending desirable.

[5]Explicitly state contingent contracts or default are ruled out by assumption.

The corresponding first order conditions are necessary and sufficient:

$$u_c(c,l)\,r = \beta E_{|\Omega}\left[u_c(c',l')\right] \tag{2}$$

$$u_c(c,l)(1+\xi) = \beta R' E_{|\Omega}\left[u_c(c',l')\right] + \lambda_h \tag{3}$$

$$0 = min\left\{\lambda_h \ , \ k'_h\right\} \tag{4}$$

$$u_c(c,l)W = u_l(c,l) \tag{5}$$

2.2 The banks

The bank is the key agent in this economy. Banks intermediate the savings from the households to the firm more efficiently than if the household invests directly, yet they are constraint by a financial friction that gives rise to a leverage constraint. It is this constraint that ultimately makes default costly for the government.

There is continuum of mass 1 of identical banks, each of which is run by a banker. (In this subsection, I drop the index b to simplify notation). The banker is a small member of the household, hence he uses the household's stochastic discount factor to value the payoff of his activities.

The banker enters each period with a portfolio of 1 period assets and liabilities (his balance sheet) chosen last period. In particular, he holds (1) k units of firm loans which pay the ex-ante fixed gross return R; (2) b units of government bonds which return 1 unit if repaid ($Rep = 1$), otherwise they return nothing ($Rep = 0$); and (3) he owes depositors d. The net cash flow of these 3 assets defines the bank's pre-dividend equity e.

$$e = Rk + bRep - d$$

From this net cash-flow, the banker chooses how much to take home to his household, or, more formally speaking, how much dividend div to pay to his shareholders. The rest he carries into the period on the balance sheet as post dividend equity $e - div$. Given the post-dividend equity, the banker next chooses his optimal portfolio of assets and liabilities: he can invest in loans to firms k' at price 1, buy government bonds b' at price q, and sell deposits d' at price r. This yields the following end-of-period balance sheet:

Assets	Liabilities
Loans k'	Deposits $r \cdot d'$
Government bonds $q \cdot b'$	Post dividend equity $e - div$

which can equivalently be expressed by the balance sheet equation:

$$qb' + k' = e - div + rd'$$

Note that, from the lenders point of view, loans by banks are different than loans by the household - think of the latter as corporate bonds or entrepreneurial self-financing. While the bank pays 1 unit to provide 1 unit of financing to the firm, the household pays $1+\xi$ to provide 1 unit of financing to the firm. This cost difference stems from the superior screening and monitoring technology that the bank possesses. Bank intermediated financing is hence more efficient, which provides an explanation for why banks exist. As mentioned before ξ is allowed to vary over the cycle. In particular I assume that ξ is related to total factor productivity (TFP) ω according to $\xi = \bar{\xi}\omega^{-\hat{\xi}}$.

Besides the balance sheet equation, the banker's choice is constrained by several other constraints. The first constraint is crucial. Following Iacoviello (2005) I assume that bankers face a commitment problem: Between periods, i.e. after having invested into loans and bonds, but before the revelation of next period TFP, bankers can abandon the bank. In that case the depositors would take over the bank, but they would only be able to recover a fraction θ of the cash-flow, since they lack the special skills to run the bank. This option gives the banker the opportunity to renegotiate the deposit contract. Assuming that the banker has all bargaining power, the banker makes depositors a take-it-or-leave-it offer: Either depositors agree to reducing the deposit repayment to a level that leaves them indifferent between accepting or rejecting the offer, or the banker will abandon the bank. Of course depositors anticipate this, which is why, *ex ante*, they will not accept any contract that is not renegotiation-proof. This translates into a leverage constraint, requiring the bank to hold a certain minimum of expected pre-dividend equity tomorrow (weighted by the depositors' expected marginal utility):

$$E_{|\Omega}\left[u_c(c')\right] d' \leq E_{|\Omega}\left[u_c(c')\theta(Rep'\, b' + R'\, k')\right]$$

Second, the banker's dividend choice is constrained below, it can not be smaller than a certain $(1-\eta)$ fraction of equity.

$$div \geq (1-\eta)e$$

This assumption, which is common in the macro literature on financial frictions[6], makes sure that the leverage constraint introduced above bites: One on hand, the banker can't simply raise new equity should he require more. On the other hand, the fact that he constantly has to payout a positive dividend ensures that he also can't overcome the equity constraint over time by accumulating retained earnings. At the same time it is not completely at odds with reality: Banks seem to have difficulties raising capital especially in times of crisis and they generally try to smooth their dividends.[7]

Third, the bank has to hold a fixed fraction of government bonds on the balance sheet:

$$\frac{b'q}{b'q+k'} = \psi$$

[6]E.g. Gertler and Kiyotaki (2010), who motivate the same assumption by retiring bankers.

[7]The constraint implies that the dividend per (book) equity ratio (also known as dividends per share) is bounded below by a fraction of the *gross* return on (book) equity. Since the *gross* return is fairly stable across the cycle, even if the *net* return varies strongly, dividends per share are fairly stable.

Fourth, next period equity must not be negative.

$$Rk \geq d$$

Finally, there is no short selling of assets.

$$b' \geq 0, \;\; k' \geq 0$$

The bank has rational expectations and maximizes the discounted value of dividends, taking the discount factor, the prices and the law of motion of the aggregate state as given. Her objective function reads

$$V_b(d, b, k, \Omega) = max_{b',k'd',div} \; div + E_{|\Omega} \left(\beta \frac{u_c(c')}{u_c(c)} \right) V_b \left(d', b', k', \Omega' \right)$$

After substituting out *div*, rearranging and abstracting from the short selling constraints, which cannot be binding in any equilibrium with banking, and the non-negativity constraint[8], the banks problem is:

$$\begin{aligned}
V_b(d, b, k, \Omega) &= max_{b',k'd'} \;\; (e + rd' - qb' + k') \, u_c(c) + \beta E_{|\Omega} \left[V_b \left(d', b', k', \Omega' \right) \right] \\
& st. \\
E_{|\Omega} \left[u_c(c') d' \right] &\leq E_{|\Omega} \left[u_c(c') \theta (Rep' \, b' + R' \, k') \right] \\
(1 - \eta) e &\leq e + rd' - qb' - k' \\
b' q &= \psi \left(b' q + k' \right) \\
\text{where } e &= Rk + bRep - d
\end{aligned}$$

The solution of the bank's dynamic problem can conveniently be characterized as I show next.

Proposition: The vector of choice variables $[b', d', k'] \in \left(\mathbb{R}_0^+ \right)^3$ is a solution to the banks problem if and only if together with the vector of multipliers $[\lambda_1, \lambda_2, \lambda_3] \in \left(\mathbb{R}_0^+ \right)^3$ it solves the system of first order conditions

$$\partial \mathcal{L} / \partial d' \;\; : \;\;\; 0 = r u_c(c) - \beta E_{|\Omega} \left[V_{be}(e', \Omega') \right] - \lambda_1 E_{|\Omega} \left[u_c(c') \right] + \lambda_2 r \tag{6}$$

$$\partial \mathcal{L} / \partial b' \;\; : \;\;\; 0 = -q u_c(c) + \beta E_{|\Omega} \left[Rep' V_{be}(e', \Omega') \right] + \lambda_1 E_{|\Omega} \left[Rep' u_c(c') \right] \theta - \lambda_2 q + \lambda_3 q (1 - \psi) \tag{7}$$

$$\partial \mathcal{L} / \partial k' \;\; : \;\;\; 0 = -u_c(c) + \beta R E_{|\Omega} \left[V_{be}(e', \Omega') \right] + \lambda_1 R' E_{|\Omega} \left[u_c(c') \right] \theta - \lambda_2 - \lambda_3 \psi \tag{8}$$

$$\partial \mathcal{L} / \partial \lambda_1 \;\; : \;\;\; 0 = min \left\{ E_{|\Omega} \left[u_c(c) \theta (Rep' \, b' + R' k') \right] - E_{|\Omega} \left[u_c(c') d' \right] \; , \; \lambda_1 \right\} \tag{9}$$

$$\partial \mathcal{L} / \partial \lambda_2 \;\; : \;\;\; 0 = min \left\{ \eta e - qb' - k' + rd' \; , \; \lambda_2 \right\} \tag{10}$$

$$\partial \mathcal{L} / \partial \lambda_3 \;\; : \;\;\; 0 = b' q - \left(b' q + k \right) \psi \tag{11}$$

[8] The model will be parametrized such that this abstraction remains irrelevant.

where $V_{be}(e',\Omega')$, the derivative of the value function wrt. equity, is given by $V_{be}(e',\Omega') = u_c(c') + \eta\lambda_2'$.

Proof: Five steps are necessary to arrive to this result. First, note that the value function of the bank $V_b(d,b,k,\Omega)$ can be summarized as a function $V_b(e,\Omega)$ of only one endogenous state variable $e = Rk + bRep - d$ since (b,d,k) only enter the problem as this linear combination. Second, the solution of the problem of the bank $[b',d',k']$ and its value function $V_b(e,\Omega)$ is linear in e. To proof this second claim, assume that $V_b(e',\Omega')$ is linear in e'. Be x a solution to the bank's problem given e and Ω. Denote the associated value function by $\tilde{V}_b(e,\Omega)$. Then we can conclude that αx is optimal given αe for any $\alpha \in \mathbb{R}^+$ since both the objective and the constraints are linear in both e and x . (If there existed $\alpha x'$ that is feasible given αe such that $\tilde{V}_b(\alpha x',\Omega) > \tilde{V}_b(\alpha x,\Omega)$ then by linearity x' would also be feasible given e and by homogeneity it would dominate x.) Therefore the solution of the banks problem - given a linear $V_b(e',\Omega')$ - is linear in e. Furthermore, by linearity the value of this solution $\tilde{V}_b(e,\Omega)$ also needs to be linear in e. Since the same reasoning applies to the value function of all the following periods, the initial assumption that $V_b(e',\Omega')$ is linear in e' must hold. Third, the first and second result together imply $V_b(e',\Omega')$ is linear in $[b',d',k']$. Fourth, given the third result it is obvious that both the constraints and the objective of the optimization problem are affine functions. Hence, the first order conditions are necessary and sufficient. Fifth, to determine the derivative of the value function w.r.t equity $V_{be}(e',\Omega')$ we can apply the envelope theorem for maximization problems with inequality constraints: $V_{be}(e',\Omega') = u_c(c') + \eta\lambda_2'$. ■

The second part of the proof furthermore implies that the distribution of equity among banks does not matter, which allows us to represent the banking sector by a representative bank. The last step furthermore implies that the marginal value of equity $V_{be}(e',\Omega')(e',\Omega')$ does not depend on e'. This allows us to eliminate all references to the value function from the FOCs and to characterize the bank's optimal choice in terms of policy functions alone, which is convenient for the computations.

2.3 The firms

There is a continuum of mass 1 of firms. The dynamic problem of the firm can be broken down to a 2 period problem. In the first period the firm borrows k_f units from the banks and households at a fixed interest rate R', which she transforms into capital for the next period. In the second period the firm hires l_f units of labor from the households and produces. The firm uses a standard Cobb-Douglas production function with capital share α and stochastic TFP ω and capital depreciates at rate δ. TFP ω follows a log-normal AR(1) process with persistence ρ and variance σ^2

$$log(\omega) = \rho log(\omega_{-1}) + \sigma\varepsilon$$

Like the bank the firm manager is part of the household, to whom any profits or losses π are rebated lump sum and whose stochastic discount factor the firm manager uses. His sequential optimization problem is

$$max_{k'_f}\ E_{|\Omega}\left[\left(\beta\frac{u_c(c')}{u_c(c)}\right)\left(\omega'\left(k'_f\right)^{\alpha}\left(\hat{l}(\omega')\right)^{1-\alpha}+(1-\delta)k'_f-R'k'_f-W(\omega')\hat{l}(\omega')'\right)\right]$$

$$s.t$$

$$\hat{l}(\omega')=argmax_{l'_f}\ \omega'\left(k'_f\right)^{\alpha}\left(l'_f\right)^{1-\alpha}+(1-\delta)k'_f-Rk'_f-W(\omega')l'_f$$

The first order conditions determining the loan rate and the wage are necessary and sufficient:

$$R'=E_{|\Omega}\left[u_c(c')\left(\omega'\alpha k_f^{\alpha-1}l_f^{1-\alpha}+(1-\delta)\right)\right]/E_{|\Omega}\left[u_c(c')\right] \tag{12}$$

$$W=\omega(1-\alpha)k_f^{\alpha}l_f^{-\alpha} \tag{13}$$

2.4 The foreign investors

The model describes a small open economy. The rest of the world is represented by perfectly competitive risk neutral deep pocket foreign investors, who demand any asset inelastically that pays their expected rate of return $1/\bar{q}$ (i.e. the world interest rate). I assume that lending to domestic private agents requires local know-how and is not possible for the foreign investors, but they can invest in the 1 period government bond. Foreign demand for government bonds, denoted by B_x, is hence given by

$$B_x\in\begin{cases}0 & if\ E_{|\Omega}Rep'/q<1/\bar{q}\\ [0,M] & if\ E_{|\Omega}Rep'/q=1/\bar{q}\end{cases} \tag{14}$$

where M is an arbitrarily large number. This upper bound is assumed to never bind in equilibrium, however it rules out Ponzi schemes.

2.5 Market clearing

This completes the discussion of the private agents' problems. We are ready to close the private economy by the corresponding market clearing conditions. From now I will denote aggregate choice variables by capital letters, i.e. $\int_o^1 x_i di=X_I$ where i is the respective agent's index and I its type. Labor market clearing requires

$$L_F=L_H \tag{15}$$

Loan and deposit market clearing implies

$$K'_F = K'_H + K'_B \tag{16}$$

$$D'_B = D'_H \tag{17}$$

Finally, bond markets clear when the total government debt issuance $\bar{B}'$ equals the total demand by foreigners and local banks

$$B'_B + B'_X = \bar{B}' \tag{18}$$

and then by Walras' law the goods markets clear as well.

This concludes the set up of the private economy. Before we move to the problem of the government, which will choose its tax, borrowing and repayment policy, it is convenient to define the notion of a private equilibrium, given a certain set of government policies $T(\Omega)$, $\bar{B}(\Omega)$, $Rep(\Omega)$.

Definition 1: Private equilibrium Given certain state dependent government policy functions $T(\Omega)$, $\bar{B}'(\Omega)$, $Rep(\Omega)$ a *stationary private equilibrium* consists of a set of state dependent policy functions $C(\Omega)$, $L_H(\Omega)$, $D'_H(\Omega)$, $K'_H(\Omega)$, $D'_B(\Omega)$, $B'_B(\Omega)$, $K'_B(\Omega)$, $K'_F(\Omega)$, $L_F(\Omega)$, $B'_X(\Omega)$ price functions $r(\Omega)$, $q(\Omega)$, $R'(\Omega)$, $W(\Omega)$ and shadow price functions $\lambda_1(\Omega)$, $\lambda_2(\Omega)$, $\lambda_3(\Omega)$, $\lambda_H(\Omega)$ such that the maximization problems of the household, the bank and the firm are solved, the foreign lenders' demand is satisfied, markets clear and such that the policy functions imply the state transition function thats underlies the agents' expectations.

Call the vector of policy, price and shadow price functions $X(\Omega)$. Denote the functional equation system over the domain Ω, which is defined by equations (2) to (18) after replacing all individual variables with the corresponding aggregate variables[9], by $\mathcal{F}(X|\Omega) = 0$. Then, since all first order conditions are necessary and sufficient, an equivalent way of defining a *stationary private equilibrium* is: $X(\Omega)$ such that $\mathcal{F}(X|\Omega) = 0$ given any $T(\Omega)$, $\bar{B}(\Omega)$, $Rep(\Omega)$.

What are the state variables in this problem? First, we need to keep track of the exogenous state variable ω. Second, we need to account for the endogenous choices of the representative household and the bank, which have an inter-temporal dimension. This encompasses the loans extended by the household K_H; the deposits issued by the bank and bought by the household, D; the loans extended by the bank K_B; the bonds bought by the bank B_B and the loan rate set ex ante R. Furthermore Ω encompasses the endogenous variable B_X, the amount of outstanding sovereign debt held abroad, because this variable influences the government policy choices, that are discussed next. The state vector is hence $\Omega = [\omega, K_H, D, B_B, K_B, R, B_X]$.

[9]This change reflects the equilibrium requirement that individual and aggregate decisions coincide. In particular the bank's, firm's and the household's choice variables need to be replaced by their aggregate counterparts, i.e. capital letters.

2.6 The government

The model economy is governed by a benevolent government, which chooses its actions such as to maximize the households' utility. It needs to finance some fixed government expenditures G and it can do so by taxing the household through the lump sum tax T[10] and issuing 1 period government bonds $\bar{B}'$, which are a promise to repay 1 unit tomorrow and are traded at price q.[11] These bonds are sold on anonymous markets, where both local banks and international investors can buy them. While it is known who buys the bonds (banks buy B'_B, foreign lenders invest B'_X), the government cannot discriminate buyers, neither at the time of issuance nor at the time of repayment. Sovereign debt is non-enforceable and the government can not commit; the government is free to choose to repay its debt ($Rep = 1$) or to fully default on its obligations ($Rep = 0$) at the beginning of each period after observing the realization of the TFP shock. In case of default there is no direct punishment: in particular foreign lenders do not refrain from lending, and no direct output costs arise. Yet I assume that following default the government effectively can not save abroad, because its foreign assets would else be seized by the creditors.[12] This gives rise to the following government budget constraint:

$$q \cdot \bar{B}' + T \geq \bar{B} \cdot Rep + G \tag{19}$$

$$B_X \geq 0$$

It is important to note that the government is the only agent who is 'big' and hence fully understands the impact of each agents decision, including its own, on the equilibrium. This means it understands how its own decisions influence the choices of households, banks, firms and foreign lenders. While no direct default costs are assumed, the government understands that defaulting erodes the banks' equity, reduces their capacity to intermediate and hence leads to lower capital and therefore less production and income for the household. This is how default becomes costly to the government - despite the absence of any direct punishment - and hence why it may choose to repay for certain regions in the state space ex post. Furthermore, the government understands that its decision how much new debt to issue influences the expectations of everyone on the probability of default tomorrow. That means the government takes into account the effect of its actions on all private variables, including the bond price. Furthermore he government understands the comittment problem and correctly anticipates how the behaviour of the private sector and the government in the future depends on the future state vector.

We can summarize the government's problem as follows in the notation common in the default literature:

[10] Following the literature, I assume lump sum taxation. This simplifies the model by avoiding a motive for distortion smoothing.

[11] Explicitly state contingent contracts are ruled out.

[12] In the optimization problem of the government below, I enforce this constraint also before default. This simplifies matters and is innocuous since in equilibrium the constraint only binds after default for the part of the parameter space that we are interested in.

$$V_G(\Omega) = max_{Rep\in\{0,1\}}\{RepV_{GR} + (1 - Rep)V_{GD}\}$$

where the value of repayment is

$$\begin{aligned}
V_{GR}(\Omega) &= max_{T,\bar{B}',X} V_H(\Omega|Rep = 1, T, \bar{B}', X) \\
& st. \\
q \cdot \bar{B}' + T &\geq \bar{B} + G \\
B'_X &\geq 0 \\
0 &= \mathcal{F}(X, T, \bar{B}'|\Omega)
\end{aligned}$$

and the value of default is

$$\begin{aligned}
V_{GD}(\Omega) &= max_{T,\bar{B}',X} V_H(\Omega|Rep = 0, T, \bar{B}', X) \\
& st. \\
q \cdot \bar{B}' + T &\geq G \\
B_X {}' &\geq 0 \\
0 &= \mathcal{F}(X, T, \bar{B}'|\Omega)
\end{aligned}$$

Note that the condition $0 = \mathcal{F}(X|\Omega)$ constrains the governments choices to be consistent with a private equilibrium.

Since there is no direct punishment for default, the value functions $V_{GR}(\Omega)$ and $V_{GD}(\Omega)$ are equivalent, and we can write the problem more concisely:

$$\begin{aligned}
V_G(\Omega) &= max_{Rep,T,\bar{B}',X} V_H(\Omega) && (20) \\
& st. \\
q \cdot \bar{B}' + T &\geq \bar{B} \cdot Rep + G && (21) \\
B_X {}' &\geq 0 && (22) \\
0 &= \mathcal{F}(X, \Omega) && (23)
\end{aligned}$$

Definition 2: Full equilibrium: A *stationary equilibrium* in this economy is defined by a set of value functions $V^H(\Omega)$, $V^G(\Omega)$, government policy functions $T(\Omega)$, $\bar{B}'(\Omega)$, $Rep(\Omega)$, private policy functions $C(\Omega)$, $L_H(\Omega)$, $D'_H(\Omega)$, $K'_H(\Omega)$, $D'_B(\Omega)$, $B'_B(\Omega)$, $K'_B(\Omega)$, $K'_F(\Omega)$, $L_F(\Omega)$, $B'_X(\Omega)$, price functions $r(\Omega)$, $q(\Omega)$, $R'(\Omega)$, $W(\Omega)$ and shadow price functions $\lambda_1(\Omega)$, $\lambda_2(\Omega)$, $\lambda_3(\Omega)$ such that the governments problem (20) is solved subject to its budget constraint (21), the non-negativity constraint (22) and the equilibrium conditions of the private economy (23). Appendix A summarizes the government problem.

Intuitively, one can think about the equilibrium as an infinitely repeated game. Each period the government moves first choosing the new debt and tax levels and whether to default, anticipating correctly the reactions of the private agents, which in turn have correct expectations about the future. Yet another way to think about the full equilibrium is a private equilibrium that is associated with a set of time-consistent (i.e. commitment free) optimal government policy functions. Notice that this equilibrium notion is coincides both with that of time consistent policy in Klein et al. (2008) and the equilibrium applied in the sovereign default literature in the tradition of Eaton and Gersovitz (1981).[13]

2.7 Discussion of key assumptions

No selective default and no bank bail outs The assumptions that the government can not choose to default selectively and can not bail out domestic banks after default are key to generate the endogenous costs of default that sustain an equilibrium with external debt. If the government could choose to default only on foreign lenders, or equivalently to inject capital selectively into domestic banks after defaulting, external default would be costless an hence always ex-post optimal. Since foreigners would anticipate that, no foreign lending could be sustained. Theoretically, both assumptions can be justified by secondary markets, as Broner et al. (2010) show. In anonymous markets, if the government planned to default selectively (or to bail some lenders out) the defaulted upon would simply sell to the exempted (bailed out). The non-selectivity of default is plausible also empirically. While there have been a few cases of selective default, they were never with respect to the *holder* but to the *currency* (Moody's (2008)) or the *legislation* (Reinhart and Rogoff (2011a)) of the bond. It seems rather hard to target the *holders* of the bond well. The empirical case for no bailouts seems a bit harder to make, given that we do see that government defaults are often accompanied by bank bailouts. I nevertheless follow the literature (e.g. Gertler and Kiyotaki (2010), Sosa-Padilla (2012), Engler and Grosse Steffen (2016)) and rule them out. This must be understood as an approximation for the fact that these rescue packages (1) typically do not fully compensate for the full default losses, (2) are politically costly[14], and (3) are subject to complicated legal constraints.

Full default In line with most of the quantitative sovereign default literature in general and the endogenous default cost literature more specifically, I assume that the government can only choose between full repayment or full default, even though this assumption is counter factual - most defaults are partial (see e.g. Sturzenegger and Zettelmeyer (2008)). Furthermore, in my model the optimal haircut ex post would generally not be 100%, because the costs of default depend on the amount that banks loose, relative to their current equity. However, giving the country full discretion over the haircut seems unrealistic as well. A realistic representation of the default process would be to assume that a government can choose to either repay fully or

[13]Nicolini et al. (2015) discusses compares this equilibrium notion to alternative ones, where the government lacks the first mover advantage, which can give rise to self-fulfilling equilbria.

[14]Political costs can arise because the population perceives that banks were "accomplices" in bringing upon the dire situation the country finds itself in or because they are provided by an external agent like the IMF who enforces some conditionality.

enter debt renegotiations. At the end of these, a proportional haircut is applied the country's debt and the remainder repaid. Yet this haircut is not simply a choice of the defaulting country, but is the outcome of a lengthy and ex-ante highly uncertain renegotiation process.[15] Given that debt renegotiation is not the focus of this paper, it is therefore reasonable to approximate this haircut as exogenous and potentially stochastic. In order to maintain comparability with virtually all the existing sovereign default literature, I choose to set the haircut to 100%. However, the mechanism of the model would not be altered if one would assume a smaller and/or stochastic haircut.

Bond holding requirement Another important assumption is that banks have to hold a certain fraction of their assets in domestic bonds. This assumption is crucial for the model, since banks would generally not find it optimal to invest into bonds that pay the world interest rate, which is generally lower than the return on domestic capital. At the same time their exposure is necessary to sustain any foreign lending. This simple constraint is intended to capture in a reduced form are all sorts of forces that induce banks to invest heavily into domestic sovereign bonds, such as financial regulation, financial repression[16] or liquidity considerations[17]. Whatever the exact nature of these forces, its is a well documented empirical fact that banks exhibit a strong home bias and are heavily exposed to domestic sovereign debt: Sosa-Padilla (2012) and Gennaioli et al. (2014) report that on average 22% or 12% respectively of the financial sector's assets in each country in their samples are domestic sovereign debt.

International lending only to the government I assume that only the government can trade financial contracts with the rest of the world. Neither households, nor banks, nor firms can invest in or borrow from the rest of the world. This is a simplifying assumption, which could generally be relaxed as long as foreign lenders have some disadvantage in direct investment. Yet it is at the same time not an implausible approximation for many less financially integrated economies, where private borrowing makes up for only a small part of the total net foreign asset position, such as Greece for example.

Riskless loans The fact that I model loans as riskless and have the firm absorb the aggregate risk may appear unusual. Many papers with banking sectors like Gertler and Kiyotaki (2010) assume that instead bank equity buffers these shock. First note that in the absence of a friction at the level of the bank, both assumptions are equivalent. Given there are frictions they are not. In the real world most of banks assets are loans, that have a flat repayment profile as long as they are not defaulted upon. I abstract from firm default both for simplicity and because I want to isolate the effects of government default. Assuming that banks carry some of the risk associated with firm investment would change the quantitative properties of the model, since

[15] See Yue (2010a) for an extension of the Arellano model explicitly modeling the renegotiation process.

[16] Notice that in particular regulation and repression could be considered policy variables as well. I abstract from these considerations. For a model with optimal financial repression see Chari et al. (2015).

[17] For a model with a liquidity motive for holding government bonds see Engler and Grosse Steffen (2013).

it would introduce a negative correlation between the innovation in TFP and bank equity and would hence reduce the degree to which default incentives increase in TFP.

Direct lending by the household In the model direct lending from the household to the firm - think of corporate bonds but also entrepreneurial self finance - is possible, however it is less efficient than intermediated lending due to the costs ξ[18]. Direct lending becomes an attractive alternative for the household only when intermediated investment is constrained and the deposit-loan spread is hence high. The idea behind this assumption is that banks have a superior screening and monitoring technology. This technological advantage explains why banks exist and is hence consistent with the fact that the financial sector intermediates a large part of investment in many economies[19]. Furthermore, allowing for an alternative, less efficient form of investment is in line with the finding that direct investment makes up for a significant part of the loss of intermediated investment during banking crisis, documented for example by Fiore and Uhlig (2015) for Europe and by Becker and Ivashina (2014) for the US. The cost ξ is allowed to vary with TFP. For positive $\hat{\xi}$ this dependence is such that the cost advantage of intermediated financing is particularly strong when the TFP is low. This reflects the idea that the screening and monitoring technology of the bank is most useful when profitable investment opportunities are scarce.

3 Financial frictions, sovereign debt, and the costs of default

This section discusses the consequences of the financial structure of the economy and its implications for the default decision.

3.1 Underinvestment due to the financial friction

In the model economy the household has two ways to investment into physical capital: either directly by lending to the firm, or indirectly by lending to the bank, which lends to the firm. (The bank also lends to the government, but abstract from that for the moment for the sake of simplicity.) Bank lending is assumed to be more efficient, because it comes free of any transaction costs, but the capacity of banks to intermediate is constrained by a leverage constraint. Whenever this constraint does not bind, competition assures that banks make no profits. But when it binds, it drives a wedge between the return on loans and the cost of deposits, even though banks behave competitively. At the same time the lower bound on dividends that banks have to respect implies a constant drain on the banks equity. To finance this drain without its equity being eroded, the bank constantly has to make profits. Assume the share η of pre-dividend equity that the banker can maximally retain is lower than the inverse of the gross

[18] Brunnermeier and Sannikov (2014) have a similar setup.

[19] With the notable exception of the US, where market based funding plays an important role.

return on loans. In this case, the return on the equity financed assets alone is not enough to finance this drain. Hence, for equity not to erode over time banks have to make profits also on deposit financed assets. These profits can only come from a spread between the deposit rate and the return on the banks assets, which in turn arises whenever the constraint binds.

Therefore, if η is small enough, as I assume, banks are *on average* constraint by both the deposit and the dividend constraint and the spread between the deposit rate and the return on assets is *on average* bigger than zero. This means that the return on intermediated investment that households perceive is lower than it actually is, whenever the constrain binds and *on average*. Households hence under-invest in intermediated loans. At the heart of this underinvestment in banks is an externality: Households only take the deposit rate into account, but not the profits of the bank, which flow back to the household lump sum. Even if they were not the recipient of these profits, they would value them in a non-atomistic setting, because they would understand that these profits allow banks to replenish their equity, which will reduce the tightness of the constraint and make them behave more competitively.

Furthermore, if the wedge or spread between the loan rate is high enough, households will find it optimal to invest directly loans, despite the costs associated with this transaction. This alternative channel of investment, which is absent from most papers on banking, has a counter intuitive effect. One might expect that it might dampen the effects of a shortage of bank equity, since it provides an alternative, albeit less efficient, channel of investment. Indeed, it does so. But at the same time it also makes the effects of the bank equity shortage worse, because it constrains the spread between deposit and loan rate above. This implies that the leverage constraint binds tighter and it slows down the re-accumulation of bank equity through retaining the profits derived from the lending spread.

3.2 A new motive for international borrowing

The fact that the household under-invests in physical capital has an interesting effect in the context of the full open economy model with a benevolent government. First note underinvestment means that even in the non-stochastic steady state the return on capital is higher than the inverse of the discount factor $1/\beta$. Second note that whenever the government borrows today to reduce taxes today, the household will want to save most of it, anticipating higher taxes tomorrow. These additional savings will, in normal times when the bank is not to tightly constraint, flow through the banking sector to the firms. They will earn a return equal to the deposit rate, which in the non-stochastic steady state is equal to $1/\beta$, plus the spread sp. At the same time the funds have a cost of $1/\bar{q}$. Hence international lending is profitable from the benevolent government's point if view, as long as $1/\beta + sp \geq 1/\bar{q}$, that is as long as the domestic household is not much more patient than the rest of the world.

This means that the existence of capital, together with a friction that prevents the return on capital to converge to the world interest rate over time, constitutes a motive for government borrowing: The government is essentially arbitraging on the difference between the domestic and the world interest rate. Or, more loosely speaking, funds from international lenders are "cheaper" than taxes.

This return-differential motive complements the consumption-smoothing motive for participating in international financial markets.[20] Yet, there is one important difference: The consumption-smoothing motive is roughly neutral across the cycle, in the sense that in good times the government wishes to save and in bad times to borrow. Conversely, the return-differential motive is not neutral across the cycle but on average calls for international borrowing, because for moderate deviations of TFP from its mean international funds are always cheap relative to domestic ones. Hence, this motive explains why the model economy is on average (and, for the calibrated version indeed most of the time) highly indebted.[21] While the return-differential motive for international borrowing is not new in general, the fact that the financial friction sustains this motive endogenously across the cycle is a novel feature of this model. It is interesting in particular in the context of the sovereign default literature, since most models of external sovereign debt have so far relied on the simplifying assumption that the borrowing country is strongly impatient (relative to the international lenders) to generate a substantial amount of debt.[22] As we will see in the quantitative part, this model works also without that assumption.

3.3 The costs of default

As I stressed before, the model I propose assumes no direct costs of default, neither the typical output costs nor exclusion. But this does not mean that default is costless. On the contrary, the model predicts seizable and long lasting costs of default.

Figure 1, which shows how the model evolves after a default in the absence of shocks, illustrates the consequences of default. Since banks are exposed to sovereign debt, they loose a significant share of their equity. This implies that the leverage constraint, which has already been binding loosely before, now binds very tightly (see panel 2 of figure 1 for the multiplier of the leverage constraint). As a consequence, banks have to reduce the size of their balance sheets, and hence the amount of loans they extend and the amount of deposits they raise. The lower demand for deposits by banks makes the deposit rate drop, while the spread between the lending and the deposit rate increases. Given the high spread, direct investment becomes profitable for the household, despite the costs associated with this form of financing. Due to these costs however, the shift to direct financing can only partially make up for the reduction

[20] Notice, that the country can smooth its consumption even in autarky by varying capital. Nevertheless, foreign investment has additional consumption-smoothing value, since its return is exogenous, unlike the return on capital.

[21] Notice furthermore, that the cycle has opposing effects on the two motives: Low TFP constitutes a reason to borrow internationally from the point of view of the consumption smoothing motive. At the same time the return differential between foreign and domestic funds decreases with TFP and hence weakens the desirability of borrowing. The sum of the two effects, together with the varying costs of default determine how the amount and riskiness of government debt varies across the cycle.

[22] With the time period being one quarter, Arellano (2008) uses a β of .953 and a net world interest rate of 1.7%, which implies a $\bar{q}$ of 0.983. Aguiar and Gopinath (2006) use 0.8 and 1%. Mendoza and Yue (2012) use 0.88 and 1%. Joo (2014) and Gornemann (2015), which to the best of my knowledge are the only other papers which consider a sovereign default model with capital, rely on similar parameter values too.

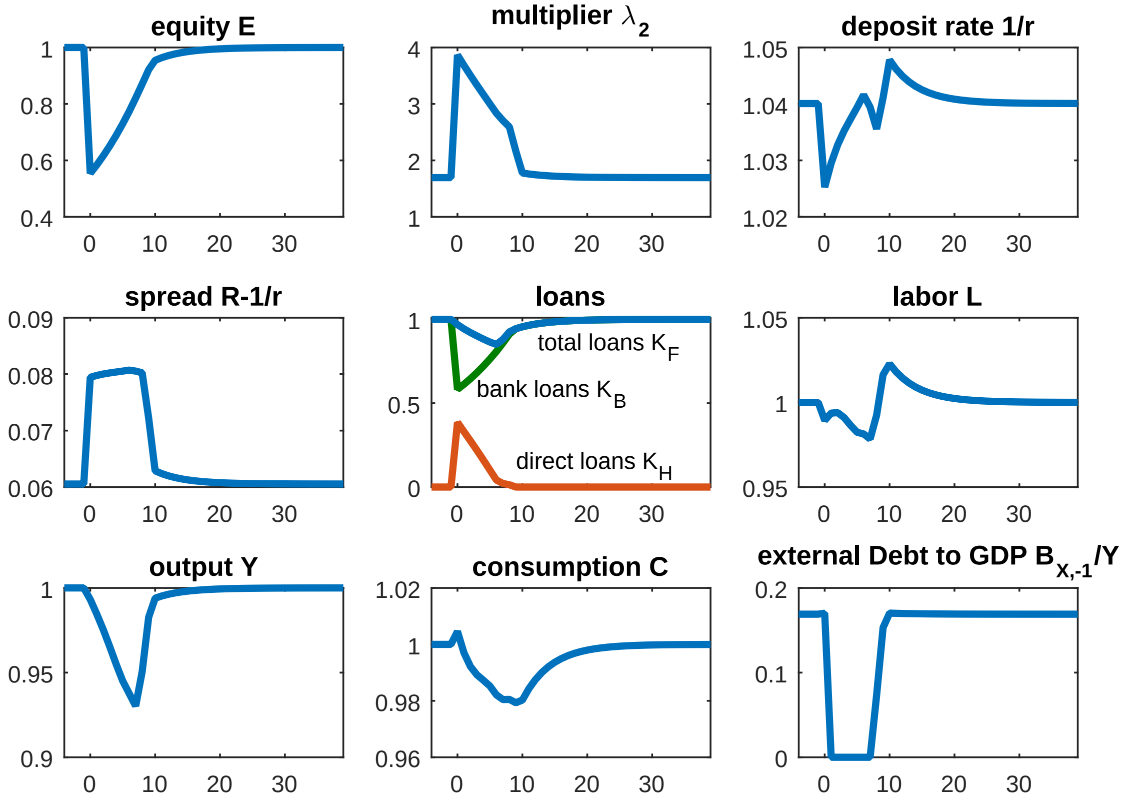

Figure 1: **Impulse responses to a default:** Prior to the default all variables are at their risk adjusted non-stochastic steady state. At period 0 the county defaults (note that this default is not optimal). No other shocks occur. Equity, capital, labor, output and consumption are normalized by their respective steady state values. Direct and bank loans are fractions of total capital. This picture is drawn for $\hat{\xi} = 0$ to make default at average TFP look more like TFP at low TFP.

in bank financing and total loans, i.e. total capital drops. At the same time the household decreases his labor supply and his consumption[23]. Since both labor and capital drop, this causes a fall in output. In the periods following default, banks slowly recapitalize by accumulating the profits generated by the higher spread on their lending activity. As they do so, they expand their balance sheet once again, and more and more of the direct investment is replaced by intermediated investment. The economy slowly converges back.

It is interesting to see what happens to external debt. After default has wiped out all foreign debt, the government does not borrow any new funds abroad. In fact they continue not to do so for several periods until they eventually enter the international capital markets again and borrow once more. What is happening? After default the government is *choosing* not to borrow any more from abroad. It does so because the main motive to borrow, which consisted in the difference between the world interest rate and the return on domestic capital, vanishes when financial intermediaries are so constrained, that the marginal return on capital is determined by the less efficient investment technology (direct investment). The government

[23]After a small initial jump in consumption due to the one-time gains from default. This jump is due to the fact that the figure plots the responses to default in the absence of shocks. In equilibrium default only occurs concurrent with bad shocks and the jump becomes negative.

essentially self-excludes itself from the market for a period of several years.[24] The duration of the period is endogenous and depends on the duration of the financial crisis.

The model is hence able to endogenously explain not only the output costs of default but also exclusion from international capital markets. While other authors have provided models that endogenize the output costs of default, they still rely on the assumption of exclusion. In the case of Mendoza and Yue (2012) exclusion is even necessary to generate the output costs. This is the first model that can also account for exclusion endogenously. Furthermore, the mechanism provides a novel explanation for *why* we observe exclusion. The argument behind the assumption of exclusion typically refers to some coordinated punishment of the borrower by the lenders. Yet this argument has one weakness: as Kletzer (1994) argues, it would be optimal for individual (small) lenders to deviate from this collective punishment strategy. The explanation for exclusion this paper proposes is immune to such criticism because it is optimal for both sides not to trade. There is simply no price at which the borrowing country and the lender would want to trade. At the same time both this and the conventional theory are observationally equivalent.

In equilibrium all agents correctly anticipate the consequences of default: They understand that defaulting on foreigners per se is a free lunch. But they also understand that defaulting on local banks causes a financial crisis. If the benefits from default are low enough, i.e. if the external debt outstanding is low, the costs of a financial crisis outweigh the benefits of default. Hence *all* debt is repaid ex post, which makes *external* debt sustainable ex ante. The financial vulnerability of the economy essentially serves as a commitment device for a government that can't commit otherwise. Notice that this entails another externality: Banks do not get remunerated for the value of the commitment that their investment in bonds entails.

3.4 Default incentives

To understand what the default decision depends on it is useful to define the default set as those states where the government defaults in equilibrium:[25] $\mathcal{D} = \{\Omega \in \Omega : V_H(Rep = 0|\Omega) > V_H(Rep = 1|\Omega)\}$. This default set has a number of features that resonate similar findings in Eaton and Gersovitz (1981) and Arellano (2008). In particular proposition 2 and the first part of the conjecture are extensions of propositions in these papers, while the other parts of the conjecture are more specific to the context of this model.

Proposition 2 Assume that $\Omega_1 = [\omega_1, K_{H1}, D_1, B_{B1}, K_{B1}, R_1, B_{X1}] \in \mathcal{D}$. Then the following holds:

$\Omega_2 = [\omega_1, K_{H1}, D_1, B_{B1}, K_{B1}, R_1, B_{X2}] \in \mathcal{D}$ if $B_{X2} \geq B_{X1}$.

Proof : First, note that the value of defaulting $V_{GD}(\Omega)$ is independent of the level of B_X, hence $V_{GD}(\Omega_1) = V_{GD}(\Omega_2)$. Second, if $\left[T_2^*, \bar{B}_2'^*, X_2^*\right]$ denotes the optimal level of all choice

[24] Notice that it might actually want to save abroad, but this is ruled out by assumption.

[25] Note that this definition does not exactly coincide with the definition in Arellano (2008), but has the same spirit.

variables under repayment given state Ω_2, then $\left[T_2^*, \bar{B}_2'^*, X_2^*\right]$ is also feasible under repayment given state Ω_1 by the government budget constraint. Hence it must be that $V_{GR}(\Omega_1) \geq V_{GR}(\Omega_2)$. Summarizing, we have $V_{GD}(\Omega_1) = V_{GD}(\Omega_2) > V_{GR}(\Omega_1) \geq V_{GR}(\Omega_2)$, and hence that $\Omega_2 \in \mathcal{D}$.■

Conjecture Assume that $\Omega_1 = [\omega_1, K_{H1}, D_1, B_{B1}, K_{B1}, R_1, B_{X1}] \in \mathcal{D}$. Then the following holds:

Part 1

$\Omega_3 = [\omega_2, K_{H1}, D_1, B_{B1}, K_{B1}, R_1, B_{X1}] \in \mathcal{D}$ if $\omega_2 \leq \omega_1$

Part 2

$\Omega_4 = [\omega_1, K_{H1}, D_2, B_{B1}, K_{B1}, R_1, B_{X1}] \in \mathcal{D}$ if $D_2 \leq D_1$

$\Omega_5 = [\omega_1, K_{H2}, D_1, B_{B1}, K_{B2}, R_1, B_{X1}] \in \mathcal{D}$ if $K_{B2} \geq K_{B1}$ and $K_{H1} + K_{B1} = K_{B2} + K_{H2}$

$\Omega_6 = [\omega_1, K_{H1}, D_1, B_{B1}, K_{B1}, R_1, B_{X1}] \in \mathcal{D}$ if $R_2 \geq R_1$

Part 3

$\Omega_7 = [\omega_1, K_{H1}, D_2, B_{B2}, K_{B1}, R_1, B_{X1}] \in \mathcal{D}$ if $B_{B2} \leq B_{B1}$ and $B_{B1} - D_1 = B_{B2} - D_2$

Proposition 3: Part 3 of the conjecture holds, if we allow the government to transfer funds from the bank's equity to the household in a lump sum fashion, but not the other way around.

Proof: First, note that conditional on default or repayment, the vector $[\omega, E, K_F, B_X]$ sufficiently summarizes the 7 dimensional state Ω (Here E denotes the pre-dividend pre-transfer bank equity $E = RK_B + RepB_B - D$). Second, note that the values of repayment are equal $V_{GR}(\Omega_1) = V_{GR}(\Omega_7)$ since the elements of the vector $[\omega, E, K_F, B_X]$ conditional on repayment are equal. Third, note that conditional on default, $[\omega, K_F, B_X]$ are equal but the banks pre-dividend pre-transfer equity is lower at Ω_1. Denote the optimal level of all choice variables under default given state Ω_1 by $\left[T_1^*, \bar{B}_1'^*, X_1^*, Z_1^*\right]$ where Z denotes the transfer from the bank to the household. Then, by choosing an appropriate transfer $Z_7 > Z_1$ the allocation $\left[T_1^*, \bar{B}_1'^*, X_1^*\right]$ must be feasible given state Ω_7. By optimality it must hence be that $V_{GD}(\Omega_7) \geq V_{GD}(\Omega_1)$. Hence $V_{GD}(\Omega_7) \geq V_{GD}(\Omega_1) \geq V_{GR}(\Omega_1) = V_{GR}(\Omega_7)$ and therefore $\Omega_7 \in \mathcal{D}$.■

Proposition 2 and conjecture state that the default set is bounded below wrt. external debt B_X, domestic debt B_B, the fraction of bank-held loans to total loans $K_B/(K_B + K_H)$ and the loan rate R and bounded above wrt. TFP ω, deposits D and the ratio bonds over equity financed bonds $B_B/(B_B - D)$. The intuition behind proposition 2 is that if default is optimal for a given level of foreign debt it must be optimal for higher levels too, because the benefit of defaulting increases but the costs remain unchanged. The conjecture is intuitive too: Part 1 states that if default is optimal for a certain level of TFP, it must be optimal for lower levels of TFP too. This is so because TFP does not affect the bank equity, but it does affect the total resources of the economy. In worse states optimal savings are lower than in good states. Hence, bank equity, which is needed to intermediate savings, is less scarce at lower levels of TFP. Hence the loss of equity due to sovereign default can be expected to be less costly at lower TFP levels. At the same time aggregate resources are more scarce at lower TFP levels, hence the benefit of not repaying the rest of the world increases. Part 2 says that default is more attractive when the bank is better capitalized, keeping the total resources of the economy and the exposure of the bank fixed. Here the idea is that on one hand the cost of default, that is the loss of bank equity, diminishes as banks are better capitalized. At the same time the benefits of default increase as banks are better capitalized, since well capitalized banks allow a less distorted allocation of the fixed amount of total resources available. Part 3 finally states that lower levels of deposit financed sovereign debt exposure of the bank make default more attractive. This is straightforward given that the post default equity of the bank decreases in the bank's deposit financed exposure.

While the conjecture cannot be proven analytically, numerical experiments have consistently confirmed it.[26]

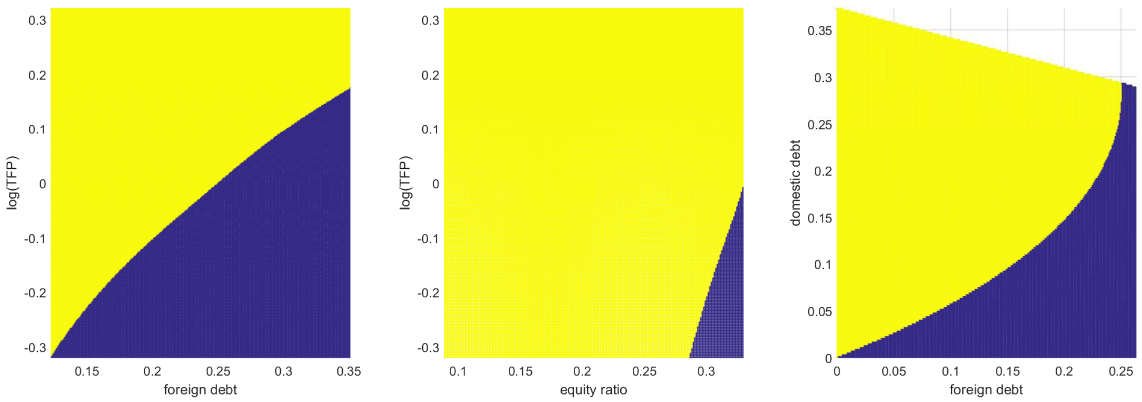

Figure 2: **Default Set:** The dark area is the default region, the bright area is the repayment region. The white area is off the equilibrium path. The remaining states are set to their non-stochastic risk-adjusted steady state values. The levels of foreign and domestic debt are normalized by the output at the non-stochastic risk adjusted steady state. The equity ratio in panel 2 is conditional on repayment.

Figure 2 shows the default set for the calibrated model, exemplifying the above characterizations. Panel 1 illustrates proposition 2 and part 1 of the conjecture: lower levels of TFP and higher levels of foreign debt make default more likely. Panel 2 illustrates the second part of the conjecture: the higher the ratio of bank equity to total resources of the economy (which may result from higher values of R, D or $K_B/(K_B+K_H)$) the more attractive default. Finally the last panel illustrates part 3 of the conjecture: The higher the domestic exposure to sovereign debt the less likely default.[27] This panel furthermore illustrates the disciplining role of domestic debt: the more exposed the domestic sector is, the more foreign debt will be repaid.

[26]Regarding part 1 of the conjecture: As the appendix shows, the numerical solution algorithm is based on the guess that part 1 holds. This guess is verified ex post at each grid point. Regarding part 3: The additional assumption necessary to proof part 3 of the conjecture does not change the equilibrium of the economy, if the value function of the government is decreasing in the share of total resources held by the bank, because then $Z^*=0$. I find that in equilibrium the value function is decreasing across the whole state space for the calibrated model.

[27]Note that here D is kept fix. That means the additional domestic debt is equity financed, so the graph actually exemplifies an even stronger conjecture that $\Omega_8=[\omega_1,K_{H1},D_1,B_{B2},K_{B1},R_1,B_{X1}]\in\mathcal{D}$ if $B_{B2}\geq B_{B1}$. For some pathological cases this stronger statement has been found to be wrong though.

While the costs of default discussed above guarantee that some amount of external debt can be sustained, they do not suffice for default to happen along the equilibrium path. For default to happen, there needs to be furthermore a strong enough desire to borrow, such that the government has an ex-ante incentive to borrow even beyond the maximal amount, for which default is ex-post undesirable. Furthermore, the default costs need to depend on the exogenous state, such that it its possible to borrow an amount that ex post leads to default only for some of the possible realizations of the exogenous shock. While the motive for borrowing has been discussed before, the state dependence of the costs of default evident in panels 1 and 2 merits further explanation.

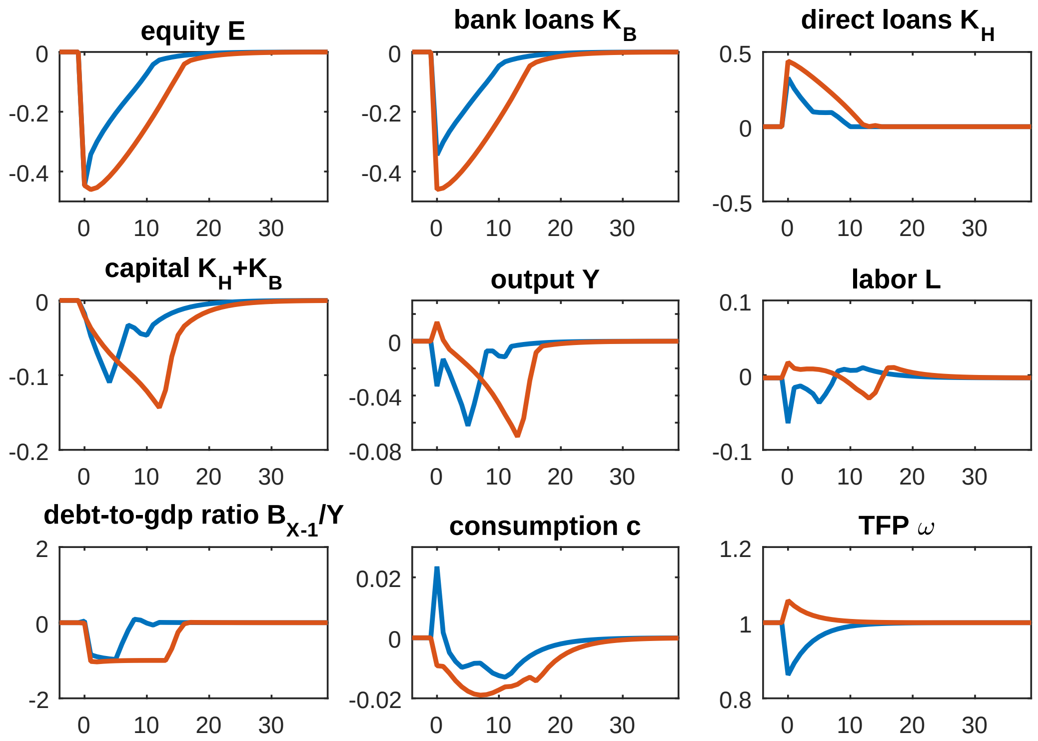

Figure 3: **Net costs of default**: This figure plots the difference between the paths associated with a two scenarios, both starting at the non-stochastic risk adjusted steady state. In scenario (1) in period 0 a TFP shock occurs and the government pays back. In scenario (2) the same TFP shock occurs at period 0 but the government defaults. No further shocks occur in either scenario. If the graph for e.g. output is negative, it hence means that (given the same TFP shock), output is lower under default than under repayment. The two lines correspond to a positive shock of one standard deviation (red line) and a negative shock big enough to trigger default (blue line). In case of the bad shock default is optimal; in case of the good shock repayment is optimal. All paths are normalized by their steady state values.

Figure 3 illustrates the dynamic difference in the net cost of default across the exogenous state. It plots the difference in the path of key variables for 2 scenarios, starting from the same initial conditions: In scenario (1) a TFP shock hits the economy in period 0 and the government repays. In scenario (2) the same TFP shock hits the economy but the government defaults. The difference between these paths illustrates the net cost/benefit of default, which consists of the sum of the downside of default (banks' equity is eroded) and the upside (the foreign debt is not being repaid). This difference is plotted for two different realizations of the TFP shock: high and low. It is evident that the net costs of default in terms of consumption and labor (and hence utility) are much higher in case of the positive TFP shock. The intuition behind this is simple. In states of high TFP the households would like to invest more (both because expected returns are high and to smooth consumption), relative to the case of a negative shock. At the same time the loss banks face due to default does not depend on the exogenous state. Hence, the fall in bank equity has stronger distortionary effects when TFP is high. This can be seen in panel 4: The total amount of loans drops much more (relative to the repayment case) under the good shock than under the bad shock. These distortions constitute the cost of default. Given that the resource gain from defaulting on the rest of the world is also independent of the state, but their utility value is higher in bad times, the net costs of default are bigger in states of high TFP.

Another subtle feature can be seen in this graph: Notice that the output costs of default in case of the bad shock, which is when default actually is optimal, increase over time. This is a result of the continued underinvestment, which results from the shortage of bank equity. As the economy under-invests in capital, the capital stock declines gradually (relative to the repayment case), which leads to a gradual decrease in output (relative to the repayment case). Hence the drop in output the results from a bad TFP shock and a simultaneous default is more persistent than the drop in output that would result from a bad TFP shock without default. This feature can help to explain the sluggishness of recoveries from default crisis.[28]

4 Computation and calibration

4.1 Computation

Given the high complexity of the model, only an approximate solution of the government's problem (20) is obtainable. Solving a continuous DSGE model numerically typically requires the uses of 3 tools: function approximation, numerical integration and numerical maximization. I briefly summarize each of these three steps here. In doing so I highlight two computational innovations that were integrated in the solution algorithm. The appendix provides more details.

I approximate the policy and value functions $Z(\Omega)$ by twice continuously differentiable functions $\tilde{Z}(\Omega)$ over the whole state space (including exogenous states). In particular I use using cubic splines defined over a multidimensional Cartesian grid as recommended by Hatchondo et al. (2007). I extrapolate points outside the grid using these splines, but choose the grid such as to ensure that the maximum distance from the grid remains small and the probability of leaving the grid marginal.

[28]Already the Arellano (2008) model has a similar feature: As the economy gradually recovers from a bad endowment shock, the default costs increase and keep endowment net of default costs depressed for longer than in case of repayment. Yet this is purely due to the recovery of the endowment. If the endowment were to stay constant for as many period as the economy remains in autarky, the default costs would remain constant too. In my model however the output costs of default increase after default, even if TFP were to remain constant.

Given this approximation for the policy and value functions, I *exactly* evaluate the integrals contained in the expectations of future values of these functions, such as $E_{|\Omega}(Z') = \int_0^\infty pdf(\omega'|\Omega)\tilde{Z}(\omega'|\Omega)d\omega$. This approach is novel and different from the quadrature approaches, that are usually used to evaluate expectations, including the application in Hatchondo et al. (2007). It extends or 'inverts' the insight of Gaussian quadrature methods, that integrals over polynomials can be evaluated exactly given a sufficient number of function values, to the piecewise-polynomial cubic-spline function. This approach has two advantages: First, it avoids an additional layer of approximation. Second, and more importantly for the current application, it generates expectations that are twice continuously differentiable not only in the endogenous states, but also in the exogenous states. This facilitates the application of continuous solution methods to optimization problems where the exogenous state appears as a variable, as it for example does when there exists a threshold value of the exogenous state, such as in the case of default models the default threshold.[29,30]

Finally, to find the stationary solution to the optimization problem of the government, given these choices for how to approximate policy and value functions and their expectations, I solve problem (20) recursively using a time iteration algorithm, that jointly iterates on the policy and value functions.[31] To find the solution at each individual time iteration, I use a numerical solver to solve the government's problem (20) continuously in all variables.

Solving the model poses some significant computational difficulties due to (1) the high dimensionality of the state space and (2) the complexity of the government maximization problem (20), which is subject to a system of nonlinear equations including complementarity conditions. To address the first problem, I use a trick that reduces the computationally necessary state space from 7 to 4 dimensions. The general idea is to replace the portfolio variables by cash-at-hand, which is only possible though after anticipating certain future decisions that depend on the whole state space.[32] This trick, which is the second methodological innovation, is explained in detail in the appendix and in its general applicability in Thaler (2016). Furthermore, the use of smooth interpolation and integration discussed above, allows me to operate with a low number of grid points without too much loss of precision. Second, to solve the complex maximization system, I rely on a modern, derivative based maximization algorithm for com-

[29] Take for example the continuous version of the Arellano model, and be $\bar{\omega}$ the default output threshold and B the debt level. Then the expectations of the value function are given by $E_{|\Omega}(V(\omega'|B')) = \int_0^{\bar{\omega}} pdf(\omega'|\omega)V^{def}(\omega'|B')d\omega + \int_{\bar{\omega}}^\infty pdf(\omega'|\omega)V^{rep}(\omega'|B')d\omega$. Therefore $E_{|\Omega}(V(\omega'|B'))$ is a function not only of the endogenous state, B' but also of the exogenous state $\bar{\omega}$. Continuous interpolation of V implies continuity of the expectation in the endogenous state. If combined with exact integration it also implies continuity of the expectation wrt. the exogenous state.

[30] This approach can be applied given any approximating function. Given a non twice-continuous approximation function, like a piecewise linear approximation, it is equivalent to straightforward quadrature, but then it does not deliver the second advantage. For chebychev polynomials, the other common twice continuous approximation function, it is equivalent to Gaussian quadrature with enough points.

[31] First make a guess for the policy and value functions $c'(\Omega)$, $\lambda_2'(\Omega)$ and $V_H'(\Omega)$. Second, I solve the optimization problem for many points on a grid over the state space Ω. Third, I update my guess given the solutions from step two and then go back to step two.

[32] Without anticipating future decisions, one can reduce the state space to 5 dimensions. Besides, by applying the same trick to the labor decision I could reduce the state by 1 further dimensions, which is left for future work.

plementarity problems (KNITRO). Given I provide algebraically computed first derivatives of target and constraint functions and the smoothness of the optimization problem, this solver is able to reliably and quickly find solutions.

4.2 Calibration

I calibrate the model to Greek data, which is the case that motivated this research. As we have seen, the model I laid out describes a small open economy whose government has access to international capital markets, with an important domestic financial sector, no foreign investment into the private sector and without the option to inflate away its debt. Greece satisfies this description to a large extent. First, Greece is a small open economy with respect to both the EU and the world. Second, the Greek government has borrowed extensively: the mean debt to GDP ratio for the post-Euro pre-default period 2000-2010 is 121% according to OECD data. This debt was held both domestically (48%) as well as abroad according to Bank of Greece data. Third, credit to the local economy is largely supplied by the domestic banking sector and non-intermediated investment plays a minor role (see e.g. Demirguc-Kunt and Levine (1999)). Similarly, domestic financial institutions account for most of the domestically held sovereign debt (80% according to Bank of Greece data). Fourth, private investment of foreigners in Greece play a minor role. As figure 4 shows, the net foreign asset position of Greece is dominated by government debt and what little net private investment there is largely flows out of Greece rather than in. Fifth, since the accession to the Euro zone, Greece can not inflate its debt away, even though they issued most of their debt under domestic currency (and law). The main difference between the model and the Greek scenario probably is the policy response: Both Greek banks as well as the government were to some extent bailed out by the EU. While the model abstracts from bailouts altogether, the bailouts that were implemented certainly did not go anywhere as far as to compensate entirely for the consequences of default.

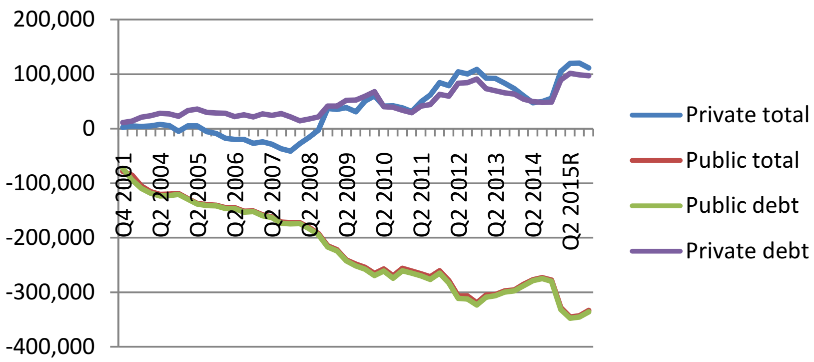

Figure 4: **International investment position:** Nominal values reported by Bank of Greece. The figure distinguishes between public and private liabilities, and between debt liabilities and total liabilities, including equity. For the government the latter two are almost identical.

While Greece seems an ideal match for the model, other southern European countries like Italy and Portugal also fit the above description. However in these cases default (resp. debt restructuring) did not materialize - even though it was a major concern at the height of the European debt crisis - which makes them less useful examples for the following quantitative analysis. Spain and Cyprus could be considered similar examples as well, the latter even defaulted in the aftermath of the Greek debt restructuring. Yet for these two countries it was a financial crisis that triggered a sovereign crisis and not the other way around. While the modeling was guided by the European debt crisis, the predictions of the model furthermore are consistent with patters documented by Gennaioli et al. (2014) across a large panel of mainly developing and emerging economies. Among these is the well studied case of the Argentinian default in 2001, where concerns regarding the damage to the financial sector indeed played a dominant role in the policy debate before and the policy response after default.[33] However, the mechanism the model describes can be expected to be stronger for countries with more developed financial systems.

The data on Greece used for the calibration and the model evaluation comes from several sources. Real GDP per capita is taken from the IMF IFS database. Its components are obtained from OECD data. TFP is taken from the AMECO database. Data on the balance sheet composition of Greek banks, the spread between short term household deposits and loans to the non-financial sector and the composition of Greek foreign liabilities is taken from the Bank of Greece. The sovereign spread is defined as the difference between the returns on one year government bonds from Greece and Germany. The data stems from the IFS for Greece and from the Bundesbank for Germany. Private credit is also taken from the IFS, while the sovereign debt to GDP ratio is taken from the OECD. For GDP and its components and TFP I use annual data covering the pre-default period 1980 to 2010, while for the financial variables the observation period is restricted to the post-Euro pre-default period 2000 to 2010. I do this both due to data availability and to account for the structural change of accessing the Euro, which should have a stronger impact on these variables. I decompose the series for GDP, TFP and credit into a linear trend and a cyclical component and use the deviation of each variable from its trend. Note that in doing so I follow Arellano (2008), who also uses a linear long-run trend, whereas many other papers using emerging economy data focus on the business cycle frequency fluctuations alone (E.g. Mendoza and Yue (2012), Aguiar and Gopinath (2006). While these differences are relatively unimportant for the much studied 2001 default of Argentina, they are significant for Greece, which experienced a much more prolonged and deep downturn, which a medium-frequency filter would attribute to the trend. Furthermore, using data only till 2010 to construct the trend, (which I then extrapolate for the post default years), prevents the estimate of the trend to be affected by the 2010 crisis. This choice is in line with my model, in which sovereign defaults are rare events that lead to large and prolonged temporary downward deviations from the trend. The trade balance is expressed in percentage of GDP.

The calibration is summarized in table 1. As in the data the time period is one year. The utility function features standard parameter values: The discount factor β is set to 0.96 while the risk aversion coefficient γ is chosen to be 2, a common value in the default literature.

[33] See Sosa-Padilla (2012), Perry and Serven (2003) and Kumhof (2004).

	Parameter	Value	Target/Explanation
α	capital share	0.36	standard value
δ	depreciation	0.14	21% investment/GDP
β	discount factor	0.96	same as β
γ	risk aversion	2	standard value
χ	labor weight in utility	1.33	mean of labor = 1
ν	inverse Frisch elasticity	0.5	standard value
G	government expenditures	0.28	21% govt. consumption/GDP
θ	financial constraint	0.7	28% equity ratio
η	share of retained equity	0.803	6% deposit-loan spread
ψ	share of bonds on balance sheet	0.14	14% exposure
$\bar{\xi}$	average cost of direct investment	0.071	1.5 ppt increase of deposit-loan
$\hat{\xi}$	dependence of cost of direct investment on GDP	0.45	spread in case of default, costs vary 2 ppt across the cycle
$\bar{q}$	inverse world interest rate	0.96	real rate Germany
ρ	persistence of TFP shock	0.76	estimate
σ	standard deviation of TFP shock	0.057	7.5% standard deviation of GDP

Table 1: **Parameter values**

The labor weight χ is set to normalize the mean of labor to 1 while ν is chosen to imply a Frisch elasticity of labor of 0.5 as in Boccola (2016), which is a common value and within the range of empirical estimates reported by Greenwood et al. (1988). The production function is parametrized in with a conventional capital share α of 36%. Depreciation δ is set such as to match the 21% average investment to GDP ratio for Greece. G is chosen to match the average government consumption to GDP ratio of 21%.

The inverse world interest rate $\bar{q}$ is set to 0.96, that is to exactly the same value as the domestic discount factor. This value is in line with the average real interest rate in (West) Germany for the period 1980 to 2010. While this choice might not seem very surprising, it is highly unusual in the context of quantitative models of sovereign default, which usually rely on a huge difference between β and $\bar{q}$ in order to generate a motive for the government to borrow. As I explained before, my model features an endogenous motive for borrowing and can therefore do without this assumption.

The choice of the financial sector parameters is the least straightforward. I choose these values in order to match moments of financial variables for Greece. Given th previous parameters, the payout parameter η is chosen to match an average spread between the loan and the deposit rate of 6%. This is in line with the average spread between 1 year deposits and firm loans for Greece. The parameters that govern the relative cost advantage of intermediated over direct investment $\bar{\xi}$ and $\hat{\xi}$ are simultaneously chosen such that direct investment becomes profitable only after an extreme shock (like default) and so as to match the 1.5ppt increase in the deposit-loan spread after default observed in the Greek data. This implies that ξ varies only mildly.[34] The share of domestic bonds on banks balance sheets ψ is set to 14%, which is the mean of the Greek credit institutions' exposure to sovereign bonds in the Bank of Greece data. Lastly, θ, which determines the tightness of the banks' (implicit) leverage constraint, is set to 0.7, which implies a mean equity ratio of 26% percent. This value, while far higher than the average for Greek banks[35], is approximately in line with the well known papers by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), which feature a leverage constraint that is very similar in spirit. Furthermore, this value guarantees that sovereign default does not drive the banks into default in the ergodic distribution. Hence the higher-than-realistic equity ratio compensates for the simplifying assumptions that default is complete and there is no partial recapitalization of banks after default.

Finally, the the persistence parameter of the TFP process ρ is estimated using detrended TFP data. The corresponding standard deviation σ is set such as to such as to match the standard deviation of detrended GDP in the data.[36] To compute the volatility of GDP for the model, following the literature, I simulate the model for 100.000 periods, extract the 31 periods prior to each default, express the data in deviation from its mean and report the mean of the volatilities of the pre-default periods.

5 Quantitative results

This section assesses the model's quantitative performance in terms of business cycle moments and the patterns observed around default episodes.

5.1 Cyclical moments

Table 2 reports the most important business cycle moments of the model and compares them to their empirical counterparts in Greek data.

The data used to generate the empirical moments has been discussed before, with the exception of the default frequency. This frequency is an empirically difficult concept: Since default is an extremely rare event, it is very hard to estimate this value well given a relatively short observation period. If one considers the longest possible observation period, that is since Greece's independence in 1829, there have been 5 default events: The 4 reported by Reinhart and Rogoff (2008) in 1843, 1860, 1893, 1932 plus the recent crisis. This yields an annual default frequency of 2.7%, which is in line with similar estimates in the literature for groups of emerging economies over the last centuries. Yet, this estimate is based on the assumption that the underlying economy has not changed over time. This is hardly the case for Greece.

[34] ξ moves from 0.06 to 0.08 as log TFP moves from 3.5 standard deviations above to 3.5 standard deviations below the mean.

[35] In the period 2000-2009 the average equity ratio for Greek credit institutions was 8.3%. In the aftermath of the crisis Greek banks have massively expanded their equity buffers, and have now reached an equity ratio of 21.3%.

[36] As commonly found in the RBC literature (e.g. Cooley and Prescott (1995)), the estimated volatility of TFP is too small for the model to explain observed volatilities. Hence, to capture the observed volatility of output in a simple model with just one shock I scale up the estimate of σ.

	Moment	Data	Model
	Means		
$E(\bar{B}/Y)$	Total debt to GDP ratio	121%	44%
$E(B_X/Y)$	Foreign debt to GDP ratio	51%	18%
$E(B_X/\bar{B})$	Share of domestic debt	47%	59%
$E(spread)$	Sovereign spread	0.48%	0.50%
$E(Rep)$	Default frequency	$< 2.7\%$	0.44%
	Volatilities		
$\sigma(Y)$	GDP	7.5%	7.4%
$\sigma(\bar{B}/Y)$	Total debt to GDP ratio	7.3%	4.2%
$\sigma(TB)$	Trade balance	3.7%	2.9%
$\sigma(spread)$	Sovereign spread	0.55%	0.26%
$\sigma(C)/\sigma(Y)$	Relative consumption volatility	79%	83%
$\sigma(I)/\sigma(Y)$	Relative investment volatility	226%	306%
	Correlations of GDP with		
$corr(Y, C)$	Consumption	95%	93%
$corr(Y, I)$	Investment	91%	81%
$corr(Y, spread)$	Sovereign spread	-28%	-70%
$corr(Y, TB)$	Trade balance	11%	-1%

Table 2: **Cyclical moments**

Financial development for example arguably has made default more costly. Given that my model describes a financially developed economy, in which the costs of defaults are presumably larger than they were in past centuries or are in economies with less developed financial markets, I consider this estimate an upper bound for the default frequency at best. The low spreads on Greek bonds in the post-Euro pre-default period seem to confirm this notion.

To generate the moments for the stochastic stationary state of the model, I simulate the model for 100,000 periods, discarding the initial observations. From this series I calculate the default frequency. Then I take the values for the 31 periods preceding each default event (excluding cases where another default happened in this period), normalize them by their mean and calculate the respective moments for each pre-default episode. Table 2 reports the means of these moments. Notice that, with the exception of the output volatility, none of these moments were targeted.

The model predicts a total debt to GDP ratio of 44% and a foreign debt to GDP ratio of 18%, which is less than half of their empirical counterparts. This is a well known property of models with short term debt. For example, Arellano (2008) and Mendoza and Yue (2012) report even lower numbers: They obtain 6% and 23% quarterly debt to quarterly GDP, or 1.5% and 5.75% debt to annual GDP. As Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) show, more realistic ratios can be obtained with long term debt, yet this extension is beyond the scope of this paper. The magnitude of the volatility of the debt to GDP ratio lies in the ballpark of what is measured in the data[37]. Furthermore the model predicts that 59% of debt are domestically held, which is not very far away from the 47% observed in the data.

Like in all models with risk neutral lenders, the default frequency and the mean of the sovereign spread coincide by assumption (almost[38]). Hence, the model can only match one of the two numbers. In fact the model matches the spread very well, which has the consequence that the predicted default frequency is much lower than the "historical upper bound estimate" discussed above. Furthermore the model yields a negative correlation between GDP and the spread, and in doing so reproduces a stylized fact that holds not only for the Greek default episode studied here, but is generally found in studies of sovereign debt (see e.g. Mendoza and Yue (2012)). At the same time the model underestimates the volatility of the spreads.

Besides, the model preserves the typical features of the real business cycle model: As in the data, consumption is slightly less volatile and investment significantly more volatile than output. Moreover, the model matches the data well in predicting a very high correlation between consumption and GDP, and, albeit less perfectly, a slight lower but still high correlation between investment and output. Furthermore the model predicts a trade balance that is about as volatile and as uncorrelated to GDP as in the data.

5.2 Default episodes

Next I study the model's dynamics around default events. This is done using event study techniques. Similarly as before, I extract the period starting 5 years before default and ending 25 years after from the simulated data and normalize these values by their respective means. I plot the mean of the evolution of each variable in figure 5. To highlight the stochastic properties of these paths, their 10th to 90th percentiles are also plotted. The figure furthermore shows the evolution of the respective variables in the data. The default period 0 corresponds to 2011 for the data.[39] GDP values beyond 2015 are IMF forecasts.

Several features are worth noticing. First, in line with the other two stylized facts documented by Mendoza and Yue (2012) the model predicts that default happens in bad times. Prior to default output is below trend 2/3 of the time and default itself is associated with a strong drop in GDP. Furthermore the external debt ratio is above average prior to default and peaks in the default period. Second, the magnitude of the output drop associated with default predicted by the model coincides with the what we observed in Greece, where GDP dropped by a quarter after default. Third, the model predicts a credit crunch: Funds supplied to the real economy drop, while the spread between the deposit and loan rate spikes. Again, the

[37]Note that this number is very poorly estimated. Excluding the last observation (2010) slashes this number by 2/3

[38]The following relationship holds $spread = \frac{P(Rep=0)}{1-P(Rep=0)}\frac{1}{\bar{q}}$, which for $\bar{q}$ and $1 - P(Rep = 0)$ close to 1 can be approximated as $spread \simeq P(Rep = 0)$. Furthermore, the default frequency refers to the whole sample and the spread to pre-default periods.

[39]The default of Greece was a somewhat gradual process. While Greece effectively had lost access to capital markets by the end of 2010, Greek banks registered their first losses on sovereign bonds in August 2011 when they participated in the securities exchange program. The process of debt restructuring was finished in April 2012. When mapping this process to the more stylized default decision in the model, it seems reasonable to choose 2011 as the default period.

magnitudes predicted by the model are similar to what we observe in the data. Fourth, while the model understates the magnitude of the drop in consumption, it does a fairly good job with respect to investment. Fifth, the responses predicted by the model are less protracted than what we observe in Greece, despite the fact, that the output costs increase over time (recall figure 3). This divergence is explained by the relatively quick reversal of TFP in the model and the linear detrending applied to the data.

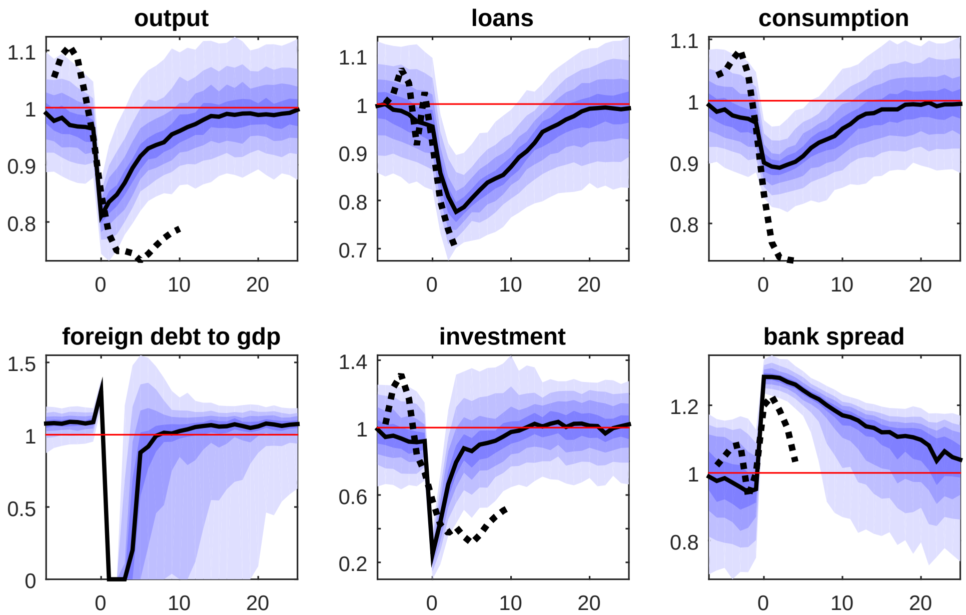

Figure 5: **Default episodes:** This figure plots the mean of the simulated path of key variables around default events (black solid line). Furthermore the 10th to 90th percentiles are plotted in blue. The black dotted line corresponds to the evolution of the variable in Greek data, where loans are equated to private credit. All variables are normalized by their means. Default occurs at period 0. For the data this period corresponds to 2011.

Most importantly, the model predicts that default is followed by a period of several years, during which the government does not issue any new debt. Unlike in existing models of sovereign default, this is an equilibrium outcome. The average duration of this period is 4 years, which is close to the values usually assumed in the default literature. With Greece still excluded from the capital markets as of summer 2016, we still have to see how long it will take till Greece re-accesses the markets. Yet the prediction of the model is well within the range of 1 to 8 years estimated in Gelos et al. (2011), Richmond and Dias (2009) and Cruces and Trebesch (2013). Furthermore, note that the endogenous duration of exclusion is stochastic. As figure 6 shows, its distribution is strongly skewed to the right, with a mean of 5.8 and a median of 4 years.[40] This property of the model is in accordance with empirical observations: Using a sample of 147 defaults since 1975 Schmitt-Grohe and Uribe (2016) report a mean duration of default of 8 and a median of 5 years. However, their sample contains a lot of developing

[40] I consider a country to have re-accessed capital markets if it borrows for two consecutive periods. This way, the exclusion periods include a few episodes where the government interrupts its no-borrowing period for one period due to a strong temporary shock.

countries. Constraining the sample to the 50% more financially developed countries according to the financial institutions development index proposed by Svirydzenka (2016)[41] reduces these values to 3 and 6.[42]

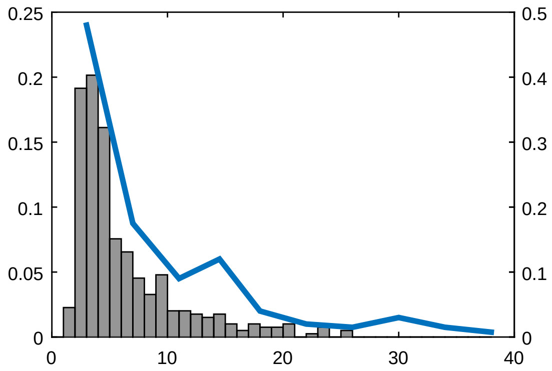

Figure 6: **Length of default episodes:** The bar histogram refers to the model, the distribution function (blue line, right axis) is copied from Schmitt-Grohe and Uribe (2016), Figure 13.1.

5.3 Robustness

The results are qualitatively robust against a number of qualitative changes. The exact shape of the financial friction for example does not influence the results much. In particular, using a state dependent leverage constraint as in Gertler and Kiyotaki (2010) or a simple leverage constraint , does not affect the results much

[43]. Furthermore, the main results are robust to using GHH preferences or abstracting from labor all together. Using government spending / endowment shocks produces similar results as

[41]Thereby I exclude defaults of most African and less developed Latin countries.

[42]Two caveats are in order: (1) Schmitt-Grohe and Uribe (2016) report the duration of default, not of non-zero trade, which need not necessarily coincide. (2) There are several plausible ways to define the start and end dates of default and market access which lead to deviations across different studies. However, the skewness seems to be a robust feature.

Gelos et al. (2011) measure the time from default to reaccess and report a mean of 4.7 and a median of 3.5 period across 45 default episodes. Combining the data from Schmitt-Grohe and Uribe (2016) to date entry into default and Cruces and Trebesch (2013) to date reaccess to the markets and restricting the set to the more developed countries leaves us with only 38 episodes and yields a median of 7.5 and a mean of 9. Among the episodes I referred to before as cases where the financial sector played an important role, the exclusion period is significantly shorter: Greece (2012) 4+, Cyprus (2013) 2, Argentina (2001) 5, Russia (1999) 2.

[43]A simple leverage constraint would require the bank to finance no more than a certain fraction of their assets by deposits: $d'r \leq \theta(k' + qb')$. As in Iacoviello (2005) I assume that the bank is required to promise no more than a certain fraction of their net return to depositors $E_{|\Omega}\left[u_c(c')\right] d' \leq \theta E_{|\Omega}\left[u_c(c')(Rep'\, b' + R'\, k')\right]$Gertler and Kiyotaki (2010) assume that the continuation value of banking has to be bigger than a certain (divertable) fraction of the assets $E_{|\Omega}\left[V_B'\right] \geq \theta(k' + q'b)$. These three different constraints have slightly different implications: For a simple leverage constraint interest rates do not matter, and book equity is constant as long as the constraint binds. For the constraint I use today's interest rates matter. The higher the spread between the loan and the deposit rate, the higher today's leverage is allowed to be. This is so because higher bank profits tomorrow increase the value of the bank in case it is taken over by depositors. The constrained proposed by Gertler and Kiyotaki (2010) extends this reasoning to the infinite future: the discounted value of the profits in all future periods disciplines the banker. Another way to look at the three constraints is to say that the first constrains the book-value leverage, the third constrains the market value leverage and the second lies in between.

well.

Furthermore, most of the conventional parameters have relatively little quantitative effect on the default statistics of the model as well. This is in particular true for the level of government spending and the parameters of the utility function and the production function. Yet, allowing the domestic economy to be impatient relative to the world interest rate, as the literature routinely does, the model yields significantly higher default rates.

The financial sector parameters play a more important role, since they determine the costs of default. The parameter ψ, which determines the exposure of banks to sovereign debt and hence the strength of the financial crisis triggered by default, increases the costs of default and hence the amount of debt which is sustainable. The intensity of the financial friction θ determines the leverage of banks. The higher the leverage, the higher the costs of default and hence the amount of sustainable debt. The dividend payout parameter η pins down the spread between the bank lending and the deposit rate. The higher the spread, the higher the difference between the steady state return on domestic capital and the world rate, hence the stronger the incentive for the government to borrow - more and at higher risk. Finally the cost advantage of intermediated investment ξ has an ambiguous effect on the default incentives. Consider first the case that ξ is constant, i.e. $\hat{\xi} = 0$. While within one period, a lower cost advantage of intermediation means that the reduced intermediation capacity of banks due to the shortage of bank equity is less harmful for the capital stock, it also means that the maximum spread which banks can charge is lower. This slows down the process of bank equity reaccumulation, and extends the period, during which bank equity is so scarce that direct intermediation is used. For the calibration I use I found that the second effect dominates the first (locally), and hence lowering ξ marginally increases both the period of zero external debt and the sustainable amount of debt. That lower ξ increases the debt capacity only locally becomes clear when one takes the value for ξ towards its lower extreme of zero. Trivially, at $\xi = 0$ banks become obsolete and no debt is sustainable. The other extreme, setting ξ to infinity, means shutting down direct investment and turns out to have similar effects: in that case upon default the economy experiences a very strong and very short financial crisis, during which investment drops dramatically and the deposit-loan spread shoots up to empirically very implausible levels, which though allow the bank to recapitalize almost fully within one period. The cost of this strong but short crisis is far smaller than the cumulative cost of the protracted crisis resulting from intermediate values of ξ and hence the amount of debt sustainable is lower. This highlights the crucial role of allowing for an imperfect substitute for bank intermediation to generate bigger and more protracted financial crisis. Allowing ξ to vary slightly with TFP makes the post-default bank lending spread and recovery path depend on TFP is such a way that, sovereign default becomes more attractive in bad times (in relative terms), and hence slightly increases the default probability (by about 0.1 ppt) and the debt to GDP ratio (by 1pt) and the exclusion time (1 period).

6 Policy application: The EU plan to reduce the exposure of banks to domestic sovereign debt

In the aftermath of the sovereign debt crisis, policy makers in the EU have begun discussing new rules to weaken the sovereign-bank nexus. One proposal being discussed is to limit the exposure of banks to domestic sovereign debt through various measures like ceilings or higher capital requirements on domestic sovereign exposure.[44] The intention of this proposal is to reduce the risk of a vicious cycle, in which problems of sovereign debt sustainability lead to losses for banks, which in turn affects output and hence government revenues negatively, and hence makes the sovereign debt burden even more unsustainable. Yet, looking at this proposal through the lens of this model, such a reform may have unintended consequences: Making banks less exposed to domestic sovereign, ceteris paribus, debt reduces the cost of default for the domestic economy. This has two implications. First, default will be more attractive *ex post*, given the same amount of external debt. Changing the rules from one day to the other may hence trigger sovereign defaults that would otherwise not have occurred. Second, the anticipation of lower default costs by international lenders *ex ante* costs means that governments will not be able to borrow as much as before. Hence, the benefits derived from international lending will shrink.[45] While the second implication resembles the reasoning in the theoretical analysis of Chari et al. (2015), the first is absent in their deterministic model.[46]

To illustrate these consequences quantitatively, I analyze the effects of a reduction of the banks exposure to sovereign debt ψ by 20%.[47] Table 3[48] shows the long run effects of this regulatory change: As as the banks exposure is reduced by 20%, the total debt to GDP ratio drops by one third and the foreign debt ratio by half. This loss of foreign debt capacity comes at a utility cost of about 0.9% in consumption equivalent.

The short run effect is equally stark: Assume the country finds itself at the non-stochastic risk adjusted steady state of the baseline economy at the end of period t. Assume furthermore that between periods t and $t+1$ ψ is changed unexpectedly, but the balance sheet of the bank is not yet affected. Then there is a 10% probability that TFP turns out so bad that the government will default at the beginning of $t+1$ because it can only partially roll over its outstanding foreign debt and prefers default to the high taxes otherwise needed. If we assume

[44]See for example "Sovereign debt rule changes threaten EU bank finances", Financial Times, 8.6.2016

[45]In the policy discussion it is sometimes argued that the reduced demand for government bonds by domestic banks may increase the interest rates of sovereign bonds as an unintended side effect. Yet this argument is simply based on lower *demand*, while the argument I make here is based on the lower *commitment value*. In fact the model abstracts from the demand argument altogether: In the model domestic borrowing by the government does not add any resources to the economy and taxation is lump sum, so lower domestic demand per se is irrelevant. Domestic borrowing is useful only because it provides the commitment to borrow externally.

[46]In their setup default is discriminatory and borrowing from banks is *directly* desirable for tax smoothing motives. In contsrast to that in my model default is non-discriminatory, and borrowing from banks is desirable *indirectly* because it generates comittment to borrow from international lenders at the (favorable) world interest rate. Furthermore there setup differs from the experiment herem in that they consider a deterministic economy where the exposure of banks is a policy variable of the government that can be adjusted each period.

[47]This number is arbitrary, as there is no concrete final proposal yet.

[48]Unlike in the previous table, these moments are unconditional.

Moment		Baseline	ψ-20%
$E(\bar{B}/Y)$	Total debt to GDP ratio	43%	29%
$E(B_X/Y)$	Foreign debt to GDP ratio	18%	9%
$E(B_X/\bar{B})$	Share of domestic debt	60%	69%
$E(spread)$	Sovereign spread	0.47%	0.65%
$E(Rep)$	Default frequency	0.44%	0.62%

Table 3: **Effect of reduction of exposure**

instead that the change in the regulation is affecting also the current balance sheet between the periods[49], the effect is even more striking: As the government wakes up the next morning, not only does it realize that it can't roll over its debt, but also that its banks are less exposed. It will default wit 86% probability. While these sudden change scenarios are arguably too stylized, it exemplifies the point that if the default probability along the transition to the lower ψ is to be contained, a gradual implementation of the reform is to be recommended.[50]

7 Conclusion

Motivated by the recent European sovereign debt crisis, this paper proposes a quantitative model of opportunistic sovereign default with endogenous costs of default. In particular, it aims at endogenizing two types of default costs: the lack of access to international capital markets and output costs. With this objective, I develop a model of sovereign debt with international lenders and an explicitly modeled domestic economy, which consists of households, banks and firms. Savings are intermediated from households to firms by banks. Since there is a friction in the intermediation process, which makes bank equity both necessary and scarce, the economy never reaches its optimal level of investment. This fact generates a strong incentive for the government to borrow on international markets. While the government might default on this debt ex post, the fact that domestic banks are also exposed to sovereign debt generates commitment: If the government chooses to default, banks suffer losses, which leads to a period of credit shortage. This translates into a reduced capital stock and output losses. Furthermore, while the financial crisis lasts, the domestic return on investment drops which discourages the government to borrow at the constant world interest rate. Hence default is followed not only by a period of depressed output, but also by a period during which the government issues no new bonds. These two consequences of default arise endogenously and make some amount of debt sustainable.

Conceptually, this paper therefore contributes to the existing literature in three ways. First, it introduces the capital shortage motive for borrowing into the sovereign debt literature. Sec-

[49]Assume that 0.2% of the government debt on the domestic bank's balance sheet is exchanged with the household for a claim of equal amount.

[50]It seems likely that a model with long term debt would predict even starker consequences.

ond, it proposes a new mechanism to explain the output costs of default, which is in line with recent experiences with the sovereign bank nexus. Third, it proposes a novel explanation for why we observe temporary breakdowns of international borrowing in the aftermath of default and for how long this period lasts, which is a question that has not been addressed by this literature before.

Quantitatively I show that the model calibrated to other moments predicts empirically plausible magnitudes of the consequences of default: Both the drop in output, investment and credit as well as the duration of market shutdown are roughly in line with the data.

This model furthermore offers insights that are relevant for a current policy debate on reducing banks exposure to domestic sovereign debt. I show that any such policy not only makes banks more resilient against sovereign debt crisis, which would be desirable per se. It also reduces the sustainability of sovereign debt, which is clearly undesirable, at least in the context of the model. Furthermore this model also contributes an explanation to why some countries like the US, Japan, or Italy seem to be able to sustain debt to GDP ratios that are higher than what other countries can sustain. It suggests that countries which have a higher share of domestically held debt and which have more leveraged financial sectors would suffer more in case of default and are hence less likely to renege on their obligations. The same argument may apply over time: The development of leveraged financial sectors across the developed world may explain the increase of debt to GDP ratios in the post war period.

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Appendix

Appendix A: The optimization problem of the government

Be X the vector of private sector variables $X = [C, L, D', K'_H, K'_B, B'_B, B'_X, K'_F, r, q, R', W, \lambda_1, \lambda_2, \lambda_3, \lambda_H]$. Then the government's problem, after substituting out some variables (HH value function, tomorrow's labor supply and dividend), expressing the foreign bond demand as a complementarity constraint and dropping a few redundant indices, is:

$$V_G(\Omega) = max_{Rep,T,\bar{B}',X} \quad \frac{C^{1-\gamma}}{1-\gamma} + \chi\frac{L^{1+\nu}}{1+\nu} + \beta E\left[V'_G\right]$$

$$s.t.$$

Government budget constraint

$$q\bar{B}' + T = \bar{B}Rep + G$$

$$B'_F \geq 0$$

Bank FOCs

$$0 = rC^{-\gamma} - \beta E\left[V'_{B\,e}\right] - \lambda_1 E\left[C'^{-\gamma}\right] + \lambda_2 r$$

$$0 = -qC^{-\gamma} + \beta E\left[Rep' V'_{B\,e}\right] + \lambda_1 E\left[Rep' C'^{-\gamma}\right]\theta - \lambda_2 q + \lambda_3 q(1-\psi)$$

$$0 = -C^{-\gamma} + \beta R' E\left[V'_{B\,e}\right] + \lambda_1 R' E\left[C'^{-\gamma}\right]\theta - \lambda_2 - \lambda_3\psi$$

$$0 = min\left\{E\left[u_c(C)\theta(Rep'\,B'_B + R'\,K'_B)\right] - E\left[C'^{-\gamma}D'\right]\ ,\ \lambda_1\right\}$$

$$0 = min\left\{\eta\left(RK_B + RepB_B - D\right) - qB'_B - K'_B + rD'\ ,\ \lambda_2\right\}$$

$$qB'_B = \left(qB'_B + K'_B\right)\psi$$

Household FOCs

$$C^{-\gamma} = \beta/rE\left[C'^{-\gamma}\right]$$

$$C^{-\gamma} = \beta R/(1+\xi)E\left[C'^{-\gamma}\right] + \lambda_h$$

$$0 = min\left\{\lambda_h\ ,\ K'_H\right\}$$

$$C^{-\gamma}W = \chi L^{\nu}$$

Firm FOC

$$R' = E\left[\left(C'^{-\gamma}\omega'\right)^{\frac{1-\alpha}{\nu+\alpha}+1}\alpha K_F'^{\alpha-1}\left[(1-\alpha)K_F'^{\alpha}/\chi\right]^{\frac{1-\alpha}{\nu+\alpha}}\right]/E\left[C'^{\gamma}\right] + (1-\delta)$$

$$W = \left[\omega(1-\alpha)K_F^{\alpha}L^{-\alpha}\right]$$

Foreign lenders' bond demand

$$0 = min\left\{q - E\left[Rep\right]\bar{q}\ ,\ B_X\right\}$$

Market clearing

$$\bar{B}' = B_B' + B_X'$$

$$K_F' = K_B' + K_H'$$

Resource constraint

$$C + K_B' + K_H'(1+\xi) + B_B'q = \bar{B}'q + \omega {K_F}^{\alpha} L^{1-\alpha} + (1-\delta)K_F - \bar{B}Rep + B_B - G$$

Notice that in this problem, the government not only needs to form expectations over tomorrows value function V_G', but also over the marginal utility of consumption $u_c(C', L') = C'^{-\gamma}$, the marginal value of bank equity $V_{B\,e}' = C'^{-\gamma} + \eta\lambda_2'$, the repayment choice Rep' and products thereof.

Appendix B: Computation

State space reduction:

The state space of the model described above is 7 dimensional $\Omega = \left[\omega, \bar{B}, K_H, D, B_B, K_B, R\right]$. To reduce the computational burden it us helpful to reduce its dimensionality. This can be done by anticipating the default decision. In the following I will explain this trick, which is an application of the method explained in Thaler (2016), in two steps.

First assume that for a given state Ω we know the default optimal decision. In that case, we can compute cash-at-hand (or pre-dividend equity) of the bank after debt repayment directly from the information contained in Ω:

$$E \equiv RK_B + RepB_B - D$$

Cash-at-hand for the household, after debt repayment (but before receiving the proceedings of new debt issuance) is given by the difference of the total resources of the economy minus bank equity:

$$W = \omega\left(K_B + K_H\right)^{\alpha} L^{1-\alpha} + (1-\delta)\left(K_B + K_H\right) - Rep(\bar{B} - B_B) - E$$

This variable contains the non-predetermined choice L though.[51] Therefore, we need to keep

[51] If L is constant it is enough to know E and W. Furthermore, given separable utility, one could apply the

track separately of

$$K_F = K_B + K_H$$

and

$$\tilde{B} = Rep\left(\bar{B} - B_B\right)$$

Once we know the three variables $\left[E, \tilde{B}, K_F\right]$, plus the exogenous state ω, we have sufficient information to solve the governments problem for all variables other than Rep, that is we can find $(T, \bar{B}', X)$.[52] Call this alternative "state vector" $\left[\omega, \tilde{\Omega}\right] = \left[\omega, \left[E, K_F, \tilde{B}\right]\right]$. Assume we solve the governments problem across this alternate state vector.

But what about the repayment decision, which after all depends on all the 7 state variables in Ω? This bring us to step two. Whenever we solve for $(T, \bar{B}', X)$ given the state $\left[\omega, \tilde{\Omega}\right]$, we determine the values of all the endogenous elements in Ω. That means, we can determine the value of tomorrow's endogenous alternative state *conditional on repayment*: $\tilde{\Omega}'|Rep'$. Furthermore assume that for each $(\bar{B}, K_H, D, B_B, K_B, R)$ there exists a threshold level of the exogenous shock $\bar{\omega}$, below which default is optimal and above which repayment is optimal. (This assumption is verified ex post.) Once we know this threshold value $\bar{\omega}$, we can determine tomorrow's endogenous state as a function of the exogenous state: $\tilde{\Omega}'(\omega') = \left[E'(\omega'), \tilde{B}(\omega'), K_F'\right]$. This is all we need to know in order to compute the expectations over future variables, given we have approximated them across the state $\left[\omega, \tilde{\Omega}\right]$. Finally, we need to ensure that we picked the right $\bar{\omega}$. We know that at $\bar{\omega}$ the government must be indifferent between default and repayment, i.e. $V\left(\bar{\omega}, \tilde{\Omega}|Rep' = 1\right)$=$V\left(\bar{\omega}, \tilde{\Omega}|Rep' = 0\right)$. To find this threshold level, we therefore augment the above optimization problem by the variable $\bar{\omega}$ and the condition $V\left(\bar{\omega}, \tilde{\Omega}|Rep' = 1\right)$=$V\left(\bar{\omega}, \tilde{\Omega}|Rep' = 0\right)$. I call this trick "anticipation of future choices" because what we essentially do here is to explicitly anticipate one of the choices that the government has to make tomorrow and incorporate it into todays problem - with the aim of being able to reduce the state space.

This means, that by complicating the optimization problem only slightly (by adding one equation and one variable), we are able to reduce the computationally necessary state space from 7D to 4D[53]. This brings about a massive reduction in computation time.

Algorithm:

Apart from the way to deal with the state space and to evaluate integrals (expectations), the time iteration algorithm I use is a standard one loop algorithm, commonly used in the sovereign default literature.

trick that I apply to the repayment decision also to L. Then, even with variable labor, knowing E, W and ω is sufficient.

[52] To see this note that the state variables only appear in the above problem in these combinations.

[53] As mentioned in the main text, a reduction to 5D is feasible without this trick, and a further reduction to 3D is feasible by applying the same trick o the labor decision.

1. Define a Cartesian grid over the 4D alternative state vector. Instead of using $\left[\omega, E, K_F, \tilde{B}\right]$ I rotate the grid so as to reduce the inclusion of regions of the state space into the grid, which are never visited along the equilibrium path. In particular I use $\left[\omega, \widetilde{W}, E/\widetilde{W}, K_F - (\varsigma_0 + \varsigma_1 W)\right]$ where $\widetilde{W} = (K_B + K_H)^{\alpha} + (1-\delta)(K_B + K_H) - Rep(\bar{B} - B_B)$ and $\varsigma_0 + \varsigma_1 \widetilde{W}$ is the result of a regression of K_F on $\widetilde{W}$. I use (8,10,8,5) points.

2. Make an initial guesses for the functions $u_c(C)$, λ_2 and V_G across the points of this grid.

3. From these guesses and this grid, construct a cubic spline interpolant for each of the 3 functions. I use not-a-knot end conditions.

4. For each of the points on the grid, solve the optimization problem described in appendix A, augmented by the additional state $\bar{\omega}$ and the additional equation discussed above.

5. Check the difference between the previous guess and the solutions at the grid points obtained. If they are very similar stop. Else update the initial guess and return to point 3.

The whole code is written in MATLAB. To solve the nonlinear optimization problem at each of the grid points at step 4 of the algorithm, I use the solver KNITRO, which is able to solve smooth complementarity problems fast and reliably. To improve the performance of the solver, I supply analytical first derivatives, which are largely computed and coded automatically using MATLAB's symbolic toolbox. Furthermore the code is executed in parallel.

Despite the fact that the policy functions exhibit minor kinks but are approximated by smooth functions, the precision of the result is satisfactory: The algorithm converges successfully up to an average (across the grid) absolute change of the forward looking variables of 0.0001%. Across a very long simulation the mean absolute error of the forward looking variables (also known as Euler Error, see Judd (1998)) is around 0.05%.