Discussion

When Central Banks Reveal Future Interest Rates: Alignment of Expectations vs. Creative Opacity

by

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EPFL and CMC
The Woodford (2003) Model

\[ \pi_t = E_t^P \pi_{t+1} + \kappa_1 y_t \]

where \( y_t \equiv \hat{Y}_t - \hat{Y}_t^n \) is the output gap

\[ y_t = E_t^P y_{t+1} - \kappa_2 (r_t - E_t \pi_{t+1}) + v_t \]

where \( v_t \equiv -(g_t - \hat{Y}_t^n) - E_t (g_{t+1} - \hat{Y}_t^n) \) is the only exogenous disturbance

\( \hat{Y}_t^n \) includes productivity and preference shocks

\( g_t \) includes government spending and preference shocks
The Model Here

\[ \pi_t = E^P_t \pi_{t+1} + \kappa_1 y_t + u_t \]

\[ y_t = E^P_t y_{t+1} - \kappa_2 (r_t - E_t \pi_{t+1}) + v_t \]

where \( u_t \) is uniformly distributed over the real line with infinite variance

What is \( u_t \)?

There are shocks in period 1 and 2; in period 3 the economy is at the steady state

With sticky prices á la Calvo (as in Woodford (2003)), convergence to the steady state takes time
\[ \begin{array}{cccccc}
E_{1r_2}^{CB} & E_1^P \pi_2 & \varepsilon_1 & \infty & \infty \\
r_1 & E_1^P \pi_2 & \pi_1 & r_2 & E_2^P \pi_2 & \pi_2 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
E_{0,1}^{CB} & E_{1,1}^{CB} & E_{1,2}^{CB} & E_{2,2}^{CB} & E_{2,2}^P \\
E_{0,1}^P & E_{1,1}^P & E_{1,2}^P & E_{2,2}^P \\
\frac{1}{\kappa \alpha} & \frac{1}{(1-\kappa) \alpha} & \frac{1}{\kappa \alpha} & \frac{1}{(1-\kappa) \alpha} \\
\frac{1}{\kappa \beta} & \frac{1}{(1-\kappa) \beta} & \frac{1}{\kappa \beta} & \frac{1}{(1-\kappa) \beta} \\
\end{array} \]
Signals

• Are signals correlated with the shocks?

• If not, why would one use such signals?
Transparency

After the signals, the central bank chooses $r_1$ and announces $E^C_1 r_2$

$$\min_{r_1} E^C_1 (\pi^2_1 + \pi^2_2)$$

s.t. $\pi_1 = (1 + \kappa)E^P_1 \pi_2 - \kappa(r_1 + E^P_1 r_2) + \epsilon_1$

$$\pi_2 = -\kappa r_2 + \epsilon_2$$

$$\epsilon_1 \equiv u_1 + \kappa_1 v_1, \epsilon_2 \equiv u_2 + \kappa_1 v_2, \kappa \equiv \kappa_1 \kappa_2$$

Solution:

$$r_1 = \frac{1}{\kappa} (E^C_1 \epsilon_1 - E^C_1 \epsilon_2), \quad E^C_1 r_2 = \frac{E^C_1 E^C_2 \epsilon_2}{\kappa} = \frac{\epsilon_{1,2}^{CB}}{\kappa}$$
P sets $E^P_1 \pi_2$ using the signal $\epsilon^{CB}_{1,2}$

$\epsilon_1, \pi_1$ are realized and the CB extracts $\epsilon^P_{1,2}$

Expectations are aligned as of the beginning of period 2

$$E^P_1 E^{CB}_2 \epsilon_2 = E^P_1 \epsilon_2$$

This implies

$$E^P_1 \pi_2 = 0$$

$$\pi_2 = (\epsilon_2 - E^{CB}_2)$$

$$\pi_1 = (\epsilon_1 - E^{CB}_1 \epsilon_1) + \frac{1}{1 + z} (\epsilon^{CB}_{1,2} - \epsilon^P_{1,2})$$
\[
\begin{align*}
\epsilon_{0,1} & \quad \epsilon_{1,1} & \quad \epsilon_{1,2} \\
\epsilon_{0,1}^P & \quad \epsilon_{1,1}^P & \quad \epsilon_{1,2}^P \\
\frac{1}{\kappa\alpha} & \quad \frac{1}{(1-\kappa)\alpha} & \quad \frac{1}{\kappa\alpha} \\
\frac{1}{\kappa\beta} & \quad \frac{1}{(1-\kappa)\beta} & \quad \frac{1}{\kappa\beta}
\end{align*}
\]
Opacity

- $r_1 = \frac{1}{\kappa}(E_1^{CB}\epsilon_1 - E_1^{CB}\epsilon_2)$ same, $E_1^{CB}r_2$ not announced
- P sets $E_1^P\pi_2$ using $r_1$ as a noisy (for $E_1^P\epsilon_2$) and biased (for $E_1^P E_2^{CB}\epsilon_2$) signal of $\epsilon_{1,2}^{CB}$
- Noisy because it involves $E_1^{CB}\epsilon_1, E_1^{CB}\epsilon_2$ and the latter is not known by P
- Biased: because P uses $E_1^P\epsilon_1$ as a signal for $E_1^{CB}\epsilon_1$
- It follows that $E_1^P E_2^{CB}\epsilon_2 \neq E_1^P\epsilon_2 \rightarrow E_1^P\pi_2 \neq 0$
Transparency

\[ \pi_1 = (\epsilon_1 - E_1^{CB} \epsilon_1) - (E_1^P \epsilon_2 - E_1^{CB} \epsilon_2) \]

\[ E_1^P \pi_2 = 0 \]

Opacity

\[ \pi_1 = (2 + \kappa)E_1^P \pi_2 + (\epsilon_1 - E_1^{CB} \epsilon_1) - (E_1^P \epsilon_2 - E_1^{CB} \epsilon_2) \]

\[ E_1^P \pi_2 = (-\gamma_1 + \gamma_2 - \gamma_3) \left[ ((\epsilon_{1,2}^{CB} - \epsilon_{1,2}^P) + (E_1^P \epsilon_1 - E_1^{CB} \epsilon_1) \right] \neq 0 \]
Comparison

CB and P have loss function

\[ L = E(\pi_1^2) + E(\pi_2^2) \]

Both under transparency and opacity, \( L \) is infinitely large because \( Var(\epsilon_1) = Var(\epsilon_2) = \infty \)

The expressions \( L^{tr} \) and \( L^{op} \) ignore \( \epsilon_1, \epsilon_2 \)

One can still look at \( L^{tr} - L^{op} \), however it is not a well-specified problem
Transparency is always superior in period 2

In period 1, opacity increases the variance of $\pi_1$ by adding the variance of $E_1^P \pi_2$ and by increasing the distance $E_1^P \epsilon_2 - E_1^{CB}$

However, $Cov(E_1^P \pi_2, E_1^P \epsilon_2 - E_1^{CB} \epsilon_2) > 0$ reduces the variance of $\pi_1$

This arises when $k$ is high, $\alpha$ low

$r_1 \uparrow$ is attributed mainly to $E_1^{CB} \epsilon_2) \downarrow$, which leads P to believe that CB expects a deflation, so that

$E_1^P r_2 \downarrow$, $E_1^P \pi_2 \uparrow$

$Cov(E_1^P \pi_2, E_1^P \epsilon_2 - E_1^{CB} \epsilon_2) = \gamma_3(\theta - 1) \frac{1-k}{k\alpha}$
Welfare

$L = E(\pi_1^2) + E(\pi_2^2)$ is the social loss function *only* if steady state output is efficient

If not, $L = E(\pi_1^2 + \lambda y_1^2) + E(\pi_2^2 + \lambda y_2^2)$ is the social loss function

Would the results be robust with a more general loss function?
Relevance

This result holds for a specific structure of signals, for low precision of CB’s signal relative to P’s signal, and for early signals being more precise than later ones.

Is this the relevant scenario?

The CB often polls P (for inflationary expectations, for example). How would this affect the results?